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ORIGINIAL ARTICLE

# Asymptotical state estimation of fuzzy cellular neural networks with time delay in the leakage term and mixed delays: Sample-data approach

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**Abstract** In this paper, the sampled measurement is used to estimate the neuron states, instead of the continuous measurement, and a sampled-data estimator is constructed. Leakage delay is used to unstable the neuron states. It is a challenging task to develop delay dependent condition to estimate the unstable neuron states through available sampled output measurements such that the error-state system is globally asymptotically stable. By constructing Lyapunov–Krasovskii functional (LKF), a sufficient condition depending on the sampling period is obtained in terms of linear matrix inequalities (LMIs). Moreover, by using the free-weighting matrices method, simple and efficient criterion is derived in terms of LMIs for estimation.

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## 1. Introduction

Cellular neural networks (CNNs), proposed by Chua and Yang in [1,2] have been extensively studied both in theory and in applications. Based on traditional CNN, the fuzzy cellular neural networks (FCNNs) have been introduced at the first time in 1996, proposed by Yang in [3,4]. The FCNN is a fuzzy neural networks which integrates fuzzy logic into the structure of traditional CNN. It is a very useful tool in image processing and pattern recognition. However, the existence of time delays may lead to the instability or bad performance of systems [5–7]. So, it is of prime importance to consider the delay effects on the dynamical behavior of systems. Recently, FCNNs with various types of delay have been widely investigated by many authors; see [8–12] and references therein. However, so far, there has been very little existing work on FCNNs with time delay in the leakage (or "forgetting") term [13–17]. In fact, time delay in the leakage term also has great impact on the dynamics of FCNNs. As pointed out by

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Gopalsamy [18], time delay in the stabilizing negative feedback term has a tendency to destabilize a system. Moreover, the time delay involving in the first term of the state variable of dynamical networks known as leakage delay on FCNNs cannot be ignored.

It is well known that the knowledge about the system state is necessary to solve many control theory problems; for example, stabilizing a system using state feedback. In most practical cases, the physical state of the system cannot be determined by direct observation. Instead, indirect effects of the internal state are observed by the way of the system outputs. A simple example is that of vehicles in a tunnel: the rates and velocities at which vehicles enter and leave the tunnel can be observed directly, but the exact state inside the tunnel can only be estimated. If a system is observable, it is possible to fully reconstruct the system state from its output measurements using the state observer. Also, the state estimation problem for neural networks has attracted some attention in the recent years, see [19–21].

For the state estimation problem, normally the periodic type constant vector is used in the existing literatures for getting the unstable behavior in the system. Without having such constant vector the given system should be stable, see for example [19–21]. In this regard there is no meaningful idea behind for designing the estimator gain matrix $H$. Motivating this reason, in this paper, leakage delay in the leakage term is used to unstable the neuron states without constant vector. On the other hand, the sampled-data control technology has developed largely as the rapid development of computer hardware. The measurements used to estimate the neuron states are sampled by samplers. Based on the sampled measurements, in this paper a sampled-data estimator is constructed. By converting the sampling period into a time-varying but bounded delay, the error dynamics of the considered FCNN is derived in terms of a differential equation with two different time-delays [22]. To the best of authors' knowledge, there was no results available in any of the existing literature dealing the state estimation for FCNNs with time delay in the leakage term, discrete and unbounded distributed delays based on sample-data.

Motivated by the above discussion, in this paper leakage delay in the leakage term is used to unstable the neuron states. It is challenging to develop delay dependent condition to estimate the unstable neuron states through available sampled output measurements such that the error-state system is globally asymptotically stable. Based on the LKF which contains a triple-integral term, an improved delay-dependent stability criterion is derived in terms of LMIs. However using the free-weighting matrices method, simple and efficient criterion is derived in terms of LMIs for estimation. Finally, numerical examples and its simulations are provided to demonstrate the effectiveness and merits of the derived result.

**Notations** $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean Space. For any matrix $A = [a_{ij}]_{n\times n}$, let $A^T$ and $A^{-1}$ denote the transpose and the inverse of $A$, respectively. $|A| = [|a_{ij}|]_{n\times n}$. Let $A > 0$ $(A < 0)$ denotes the positive-definite (negative-definite) symmetric matrix, respectively. $I$ denotes the identity matrix of appropriate dimension. $\Lambda = \{1, 2, \ldots, n\}$ and $\Xi = \{1, 2, \ldots, m\}$. $*$ denotes the symmetric terms in a symmetric matrix.

## 2. Model formulation and preliminaries

Consider the following FCNNs with leakage delay, discrete and unbounded distributed delays

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t-\sigma) + \sum_{j=1}^{n} b_{0_{ij}} g_j(x_j(t)) \\ \quad + \sum_{j=1}^{n} b_{1_{ij}} g_j(x_j(t-\tau_1(t))) + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds \\ \quad + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) g_j(x_j(s)) ds, \quad i \in \Lambda, \\ x_i(s) = u_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{1}$$

where $u_i(\cdot) \in C((-\infty, 0], \mathbb{R})$; $\alpha_{ij}$ and $\beta_{ij}$ are the elements of fuzzy feedback MIN template, fuzzy feedback MAX template, respectively; $b_{0_{ij}}$ and $b_{1_{ij}}$ are the elements of feedback template; $\bigwedge$, $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; $x_i$ denotes the state of the $i$th neuron; $a_i$ is a diagonal matrix, $a_i$ represents the rates with which the $i$th neuron will reset their potential to the resting state in isolation when disconnected from the networks and external inputs; $g_j$ represents the neuron activation function; $k_i(s) \geqslant 0$ is the feedback kernel and satisfies

$$\int_0^{\infty} k_i(s) ds = 1, \quad i \in \Lambda. \tag{2}$$

$(A_1)$ The transmission delay $\tau_1(t)$ is a time varying delay, and it satisfies $0 \leqslant \tau_1(t) \leqslant \tau_1$, where $\tau_1$ is a positive constant;

$(A_2)$ The leakage delay satisfies $\sigma \geqslant 0$. Also, it is assumed that the neuron activation function $g(\cdot)$ satisfies the following Lipschitz condition

$$|g(x) - g(y)| \leqslant |L(x - y)|, \tag{3}$$

where $L \in \mathbb{R}^{n\times n}$ is a known constant matrix.

Our aim in this paper is to investigate an efficient estimation algorithm in order to observe the neuron states from the available network outputs. Therefore, the network measurements are assumed to satisfy $y_l(t) = c_{lj} x_i(t)$, $l \in \Xi$, $i, j \in \Lambda$, where $y_l \in \mathbb{R}^m$ is the measurement output of the $1^{\text{th}}$ neuron and $c_{lj}$ is the element of a known constant matrix with appropriate dimension.

In this paper, the measurement output is sampled before it enters the estimator. The sampled measurement is assumed to be generalized by a zero-order hold function with a sequence of hold times $0 = t_0 < t_1 < \cdots < t_k < \cdots$

$$y_l(t_k) = c_{lj} x_i(t_k), \quad t_k \leqslant t < t_{k+1}, \tag{4}$$

where $t_k$ denotes the sampling instant and satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots < \lim_{k\to+\infty} t_k = +\infty$. Moreover, the sampling period under consideration is assumed to be bounded by a known constant $\tau_2$, that is $t_{k+1} - t_k \leqslant \tau_2$ for $k \geqslant 0$.

The full order state estimation of system (1) is given as follows