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**ORIGINAL ARTICLE**

# Unsteady flow of a Maxwell fluid over a stretching surface in presence of chemical reaction

**Swati Mukhopadhyay *, Krishnendu Bhattacharyya**

*Department of Mathematics, The University of Burdwan, Burdwan 713 104, WB, India*

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**Abstract** An analysis is presented for unsteady two-dimensional flow of a Maxwell fluid over a stretching surface in presence of a first order constructive/destructive chemical reaction. Using suitable transformations, the governing partial differential equations are converted to ordinary one and are then solved numerically by shooting method. The flow fields and mass transfer are significantly influenced by the governing parameters. Fluid velocity initially decreases with increasing unsteadiness parameter and concentration decreases significantly due to unsteadiness. The effect of increasing values of the Maxwell parameter is to suppress the velocity field. But the concentration is enhanced with increasing Maxwell parameter.


## 1. Introduction

During last few years the boundary layer flow behaviours of different types of fluids attracted the interest of many researchers (Hayat et al. [1]). Due to engineering applications, the boundary layer flows of non-Newtonian fluids have been given considerable attention in the recent years. The flow characteristics of non-Newtonian fluids are quite different in comparison to Newtonian fluids. In order to obtain a clear idea of non-Newtonian fluids and their various applications, it is necessary to study their flow behaviour. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. Several models have been suggested (Hayat et al. [2,3]). Among these, the vast majority of non-Newtonian fluid models are concerned with the simple models viz. the power law and grade two or three [4–14]. These simple fluid models have some drawbacks that they are unable to provide results not having accordance with fluid flows in reality. The power-law model is used in modelling fluids with shear-dependent viscosity. But it cannot predict the effects of elasticity. On the other hand, though the fluids of grade two or three can calculate the effects of elasticity, the viscosity in these models is not shear dependent (Hayat et al. [15,16]). Moreover, they are unable to predict the effects of stress relaxation. Maxwell model, a subclass of rate type fluids, can predict the stress relaxation and therefore, have become more popular (Sadeghy et al. [17], Abel et al. [18]). This model excludes the complicating effects of shear-dependent viscosity from any boundary layer

* Corresponding author. Tel.: +91 342 2557741; fax: +91 342 2530452.
E-mail address: swati_bumath@yahoo.co.in (S. Mukhopadhyay).
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**Nomenclature**

| | |
|---|---|
| $C$ | concentration of the species of the fluid |
| $C_w$ | concentration of the wall of the surface |
| $C_\infty$ | free-stream concentration |
| $D$ | diffusion coefficient of the diffusing species |
| $f$ | non-dimensional stream function |
| $f'$ | first order derivative with respect to $\eta$ |
| $f''$ | second order derivative with respect to $\eta$ |
| $f'''$ | third order derivative with respect to $\eta$ |
| $k_1$ | reaction rate |
| $M$ | unsteadiness parameter |
| Sc | Schmidt number |
| $u$, $\upsilon$ | components of velocity in $x$ and $y$ directions |

*Greek symbols*

| | |
|---|---|
| $\beta$ | Maxwell parameter |
| $\eta$ | similarity variable |
| $\gamma$ | reaction rate parameter |
| $\lambda$ | relaxation time of the period |
| $v$ | kinematic viscosity |
| $\psi$ | stream function |
| $\phi$ | non-dimensional concentration |
| $\phi'$ | first order derivative with respect to $\eta$ |
| $\phi''$ | second order derivative with respect to $\eta$ |

analysis and enables one to focus solely on the effects of a fluid's elasticity on the characteristics of its boundary layer (Heyhat and Khabazi [19]). Hayat et al. [20] constructed an analytic solution for unsteady MHD flow in a rotating Maxwell fluid through a porous medium. Hayat et al. [21] studied the MHD flow of a UCM fluid over a porous stretching plate with the homotopy analysis method.

Mass transfer phenomenon is used in various scientific disciplines for different systems and mechanisms that involve molecular and convective transport of atoms and molecules.

The driving force for mass transfer is the difference in concentration (Hayat et al. [15]).

Cortell [22] discussed mass transfer with chemically reactive species for two classes of viscoelastic fluid over a porous stretching sheet. The transport of mass and momentum with chemical reactive species in the flow caused by a linear stretching sheet was discussed by Andersson et al. [23].

In all these above studies, the flow, temperature and concentration fields were considered to be at steady state. However, in some cases the flow field, heat, and mass transfer can be unsteady due to a sudden stretching of the flat sheet. When the surface is impulsively stretched with certain velocity, the inviscid flow is developed instantaneously. However, the flow in the viscous layer near the sheet is developed slowly, and it becomes a fully developed steady flow after a certain instant of time. Many authors [24–35] studied the problem for unsteady stretching surface under different conditions by using a similarity method to transform governing time-dependent boundary layer equations into a set of ordinary differential equations. Recently, Mukhopadhyay [36] analyzed the combined effects of slip and suction/blowing on unsteady mixed convection flow past a stretching sheet.

No attempt has been made so far to analyze the Maxwell fluid flow and mass transfer past an unsteady stretching surface in presence of first order constructive/destructive chemical reaction. The present work aims to fill the gap in the existing literature. With the help of suitable transformations the governing partial differential equations are converted to ordinary one and the reduced ordinary differential equations are solved numerically using shooting method. The effects of governing parameters on velocity and concentration fields are investigated and analysed with the help of their graphical representations.

## 2. Equations of motion

We consider laminar boundary-layer two-dimensional flow and mass transfer of an incompressible non-Newtonian Maxwell fluid over an unsteady stretching sheet. Let $C_w$ be the concentration at the sheet surface and the concentration far away from the sheet is $C_\infty$. Also the reaction of the species be the first order homogeneous chemical reaction of rate $k_1$ which varies with time.

We assume that for time $t < 0$ the fluid and mass flows are steady. The unsteady fluid and mass flows start at $t = 0$. The sheet emerges out of a slit at origin ($x = 0, y = 0$) and moves with non-uniform velocity $U(x,t) = \frac{bx}{1-\alpha t}$ where $b$,$\alpha$ are positive constants with dimensions (time) $^{-1}$, $b$ is the initial stretching rate and $\frac{b}{1-\alpha t}$ is the effective stretching rate which is increasing with time. In case of polymer extrusion, the material properties of the extruded sheet may vary with time.

The governing equations of such type of flow (Alizadeh-Pahlavan and Sadeghy [37]) and mass transfer are, in the usual notation,

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + \upsilon^2\frac{\partial^2 u}{\partial y^2} + 2u\upsilon\frac{\partial^2 u}{\partial x \partial y}\right) = v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + \upsilon\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1(C - C_\infty). \tag{3}$$

For Maxwell fluid, no unsteady term for the shear stress need to be included as mentioned by Alizadeh-Pahlavan and Sadeghy [37]. So no unsteady term appears in the coefficient of $\lambda$.

Here $u$ and $\upsilon$ are the components of velocity respectively in the $x$ and $y$ directions, $v$ is the kinematic viscosity of the fluid, $C$ is the concentration of the species of the fluid, $D$ is the diffusion coefficient of the diffusing species in the fluid, $k_1(t) = \frac{k_0}{1-\alpha t}$ is the time-dependent reaction rate; $k_1 > 0$ stands for destructive reaction whereas $k_1 < 0$ stands for constructive reaction, $k_0$ is a constant, $\lambda = \lambda_0(1 - \alpha t)$ is the relaxation time of the period, $\lambda_0$ is a constant.