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ORIGINAL ARTICLE

A note on soft connectedness

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Abstract Soft topological spaces based on soft set theory which is a collection of information granules is the mathematical formulation of approximate reasoning about information systems. In this paper, we define and explore the properties and characterizations of soft connected spaces in soft topological spaces. We expect that the findings in this paper can be promoted to the further study on soft topology to carry out general framework for the practical life applications.

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1. Introduction

The researchers introduced the concept of soft sets to deal with uncertainty and to solve complicated problems in economics, engineering, medicines, sociology and environment because of unsuccessful use of classical methods. The well known theories which can be considered as a mathematical tools for dealing with uncertainties and imperfect knowledge are: theory of fuzzy sets [1], theory of intuitionists fuzzy sets [2], theory of vague sets, theory of interval mathematics [3], theory of rough sets and theory of probability [4,5]. All these tools require the pre specification of some parameter to start with.

In 1999 Molodtsov [6] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling the problems with incomplete information. In [7], he applied successfully in directions such as, smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement. Maji et. al [8,9] gave first practical application of soft sets in decision making problems.

Many researchers have contributed toward the algebraic structures of soft set theory [10–28]. Shabir and Naz [27] initiated the study of soft topological spaces. They defined basic notions of soft topological spaces such as soft open and soft closed sets, soft subspace, soft closure, soft neighborhood of a point, soft T_i-spaces, for $i = 1, 2, 3, 4$, soft regular spaces, soft normal spaces and established their several properties. In 2011, S. Hussain and B. Ahmad [29] continued investigating the properties of soft open(closed), soft neighborhood and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary. Also in 2012, B. Ahmad and S. Hussain [30] explored the structures of soft topology using soft points. A. Kharral and B. Ahmad [31], defined and discussed the several properties of soft images and soft inverse images of soft sets. They also applied these notions to the problem of medical diagnosis in medical systems. In [32], I. Zorlutana et.al defined and discussed soft pu-continuous mappings. In [33], S. Hussain further established the fundamental and important characterizations of soft pu-continuous func-

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tions, soft pu-open functions and soft pu-closed functions via soft interior, soft closure, soft boundary and soft derived set.

2. Preliminaries

First we recall some definitions and results.

Definition 1 [6]. Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a nonempty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2 ([9,14]). For two soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is a soft subset of (G, B), if

(1) $A \subseteq B$ and
(2) for all $e \in A, F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \tilde{\subseteq} (G, B)$.

(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supset}(G, B)$.

Definition 3 [9]. Two soft sets (F, A) and (G, B) over a common universe X are said to be soft equal, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 4 [6]. The union of two soft sets of (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 5 [6]. The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X, denoted $(F, A)\tilde{\cap}(G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e)\tilde{\cap}G(e)$, for all $e \in C$.

Definition 6 [27]. The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E)\tilde{\setminus}(G, E)$, is defined as $H(e) = F(e) \setminus G(e)$, for all $e \in E$.

Definition 7 [27]. Let (F, E) be a soft set over X and Y be a nonempty subset of X. Then the sub soft set of (F, E) over Y denoted by (Y_F, E), is defined as follows: $F_Y(\alpha) = Y\tilde{\cap}F(\alpha)$, for all $\alpha \in E$. In other words $(Y_F, E) = \tilde{Y}\tilde{\cap}(F, E)$.

Definition 8 [27]. The relative complement of a soft set (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', A)$, where $F' : A \to P(U)$ is a mapping given by $F'(\alpha) = U\tilde{\setminus}F(\alpha)$, for all $\alpha \in A$.

Definition 9 [27]. Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X, if

(1) Φ, $\tilde{X}$ belong to τ.
(2) the union of any number of soft sets in τ belongs to τ.
(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 10 [27]. Let (X, τ, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ.

Definition 11 [29]. Let (X, τ, E) be a soft topological space over $X, (G, E)$ be a soft set over X and $x \in X$. Then (G, E) is said to be a soft neighborhood of x, if there exists a soft open set (F, E) such that $x \in (F, E)\tilde{\subset}(G, E)$.

Definition 12 [27]. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of (F, E). Clearly $\overline{(F, E)}$ is the smallest soft closed set over X which contains (F, E).

Definition 13 [32]. A soft set (F, E) over X is said to be an absolute soft set, denoted by $\tilde{X}$, if for all $e \in E$, $F(e) = \tilde{X}$. Clearly, $\tilde{X}^c = \Phi_E$ and $\Phi_E^c = \tilde{X}$.

Here we consider only soft sets (F, E) over a universe X in which all the parameters of set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 14 [32]. The soft set $(F, E) \in SS(X)_E$ is called a soft point in $\tilde{X}$, denoted by e_F, if for the element $e \in E, F(e) \neq \phi$ and $F(e^c) = \phi$ for all $e^c \in E \setminus \{e\}$.

Definition 15 [32]. The soft point e_F is said to be in the soft set (G, E), denoted by $e_F\tilde{\in}(G, E)$, if for the element $e \in E$ and $F(e) \tilde{\subseteq} G(e)$.

Definition 16 [32]. A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood (briefly: soft nbd) of the soft point $e_F \in X$, if there exists a soft open set (H, E) such that $e_F \in (H, E) \tilde{\subseteq} (G, E)$.

The soft neighborhood system of a soft point e_F, denoted by $N_\tau(e_F)$, is the family of all its soft neighborhoods.

Definition 17 [29]. Let (X, τ, E) be a soft topological space and let (G, E) be a soft set over X. The soft interior of soft set (F, E) over X denoted by $(F, E)^\circ$ and is defined as the union of all soft open sets contained in (F, E). Thus $(F, E)^\circ$ is the largest soft open set contained in (F, E).

Definition 18 [29]. Let (X, τ, E) be a soft topological space over X. Then soft boundary of soft set (F, E) over X is denoted by $\underline{(F, E)}$ and is defined as

$\underline{(F, E)} = \overline{(F, E)}\tilde{\cap}(\overline{(F, E)'})$. Obviously $\underline{(F, E)}$ is a smallest soft closed set over X containing (F, E).

Definition 19 [29]. (X, τ, E) be a soft topological space over X and Y be a nonempty subset of X. Then $\tau_Y = \{(Y_F, E) | (F, E) \in \tau\}$ is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).