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ORIGINAL ARTICLE

Existence, uniqueness and stability of random impulsive neutral partial differential equations

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Abstract In this paper, the existence, uniqueness and stability via continuous dependence of mild solution of neutral partial differential equations with random impulses are studied under sufficient condition via fixed point theory.

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1. Introduction

Neutral differential equations arise in many area of science and engineering have received much attention in the last decades. The ordinary neutral differential equation is very extensive to study the theory of aeroelasticity and the lossless transmission lines [1] and the references therein. Neutral partial differential equations with delays are motivated from stabilization of lumped control systems, theory of heat conduction in materials [2,3] and the references therein. Hernández and O'Regan [4], studied some neutral partial differential equations by assuming some temporal and spatial regularity type condition.

Recently impulsive differential equations are well to model problems see [5,6]. There is much notice in the field of fixed impulsive type equations [2,7] and the references therein. When the impulses are exist at random, the solutions of the equation behave as a stochastic process. It is quite different from deterministic impulsive differential equations and stochastic differential equations (SDEs). Iwankievicz and Nielsen [8], investigated dynamic response of non-linear systems to poisson distributed random impulses. Wu and Meng [9], first gave the general random impulsive ordinary differential equations and investigated boundedness of solutions to these models by Liapunov's direct method. Wu et al. [10–12], have studied some qualitative properties of differential equations with random impulses. In [13], the author studied the existence and exponential stability for random impulsive semilinear functional differential equations through the fixed point technique under non-uniqueness. The existence, uniqueness and stability results were discussed in [14] through Banach fixed point method for the

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system of differential equations with random impulsive effect. The author [15], studied the existence results for the random impulsive neutral functional differential equations with delays. In [16], the author studied existence results of random impulsive neutral non-autonomous differential inclusions with delays via Dhage's fixed point theorem. In [17], random impulsive semilinear functional differential inclusions were studied using the Martelli fixed point theorem and the fixed point theorem due to Covitz and Nadler. In [18], the authors generalized the distribution of random impulses with the Erlang distribution. Further we refer [19–22]. Motivated by the above mentioned works, the main purpose of this paper is to study the random impulsive neutral partial differential equations (RINDEs) to fill the gap in the above works. Aeroelasticity and the lossless transmission lines can also be modeled in the form of RINDEs.

This paper is organized as follows. In Section 2, we recall briefly the notations, definitions, lemmas and preliminaries which are used throughout this paper. In Section 3, we study the existence and uniqueness of the RINDEs by relaxing the linear growth conditions. In Section 4, we study the stability through continuous dependence on the initial values of the RINDEs. Finally in Section 5, an example is presented to illustrate our results.

2. Preliminaries

Let X be a real separable Hilbert space and Ω a nonempty set. Assume that τ_k is a random variable defined from Ω to $D_k \overset{def.}{=} (0, d_k)$ for $k = 1, 2, \ldots$, where $0 < d_k < +\infty$. Furthermore, assume that τ_k follow Erlang distribution, where $k = 1, 2, \ldots$ and let τ_i and τ_j are independent with each other as $i \neq j$ for $i, j = 1, 2, \ldots$. For the sake of simplicity, we denote $\mathfrak{R}^+ = [0, +\infty)$.

We consider neutral partial differential equations with random impulses of the form

$$\frac{d}{dt}[x(t) + g(t, x_t)] = Ax(t) + f(t, x_t), \quad t \neq \xi_k, \ t \geqslant 0, \tag{2.1}$$

$$x(\xi_k) = b_k(\tau_k)x(\xi_k^-), \quad k = 1, 2, \ldots, \tag{2.2}$$

$$x_{t_0} = \varphi, \tag{2.3}$$

where A is the infinitesimal generator of an analytic semigroup of bounded linear operators $\{S(t); t \geqslant 0\}$ with $D(A) \subset X$. If $S(t)$ is uniformly bounded analytic semigroup such that $0 \in \rho(A)$, then it is possible to define the fractional power A^η, for $0 < \eta \leqslant 1$, as a closed linear operator with dense domain $D(A^\eta)$ in X. If X_η represents the space $D(A^\eta)$ endowed with norm $\|\cdot\|$, then we have

Lemma 2.1 [21]. *Assume that the following conditions hold:*

(*i*) *For* $0 < \eta \leqslant 1$, X_η *is a Banach space.*
(*ii*) *For* $0 < \eta \leqslant \beta \leqslant 1$, *the embedding* $X_\beta \hookrightarrow X_\eta$ *is continuous.*
(*iii*) *There exists a constant* $C_\eta > 0$ *depending on* $0 < \eta \leqslant 1$ *such that*

$$\|A^\eta S(t)\|^2 \leqslant \frac{C_\eta}{t^{2\eta}}, \ t > 0.$$

Now we make the system (2.1), (2.2) and (2.3) precious: The functional $g : \mathfrak{R}^+ \times \widehat{C} \to X; f : \mathfrak{R}^+ \times \widehat{C} \to X$, $\widehat{C} = \widehat{C}((-\infty, 0], X_\eta)$ is the set of piecewise continuous functions with left-hand limit φ from $(-\infty, 0]$ into X_η. The phase space $\widehat{C}((-\infty, 0], X_\eta)$ is assumed to be equipped with the norm $\|\varphi\|_t = \sup_{-\infty<\theta\leqslant 0}|\varphi(\theta)|$. x_t is a function defined by $x_t(s) = x(t+s)$ for all $s \in (-\infty, 0]$ and fixed $t; \xi_0 = t_0$ and $\xi_k = \xi_{k-1} + \tau_k$ for $k = 1, 2, \ldots$, here $t_0 \in \mathfrak{R}^+$ is arbitrary given real number. Obviously, $t_0 = \xi_0 < \xi_1 < \xi_2 < \cdots < \lim_{k\to\infty}\xi_k = \infty$; $b_k : D_k \to X$ for each $k = 1, 2, \ldots; x(\xi_k^-) = \lim_{t\uparrow\xi_k} x(t)$ according to their paths with the norm $\|x\|_t = \sup_{-\infty<s\leqslant t}|x(s)|$ for each t satisfying $t \geqslant 0$ and $T \in \mathfrak{R}^+$ is a given number, $\|\cdot\|$ is any given norm in X_η.

Denote $\{B_t, t \geqslant 0\}$ the simple counting process generated by $\{\xi_n\}$, that is, $\{B_t \geqslant n\} = \{\xi_n \leqslant t\}$, and denote $\mathcal{F}_t$ the σ-algebra generated by $\{B_t, t \geqslant 0\}$. Then $(\Omega, P, \{\mathcal{F}_t\})$ is a probability space. Let $L_2 = L_2(\Omega, \mathcal{F}_t, X)$ denote the Hilbert space of all $\mathcal{F}_t$-measurable square integrable random variables with values in X.

Let $\mathcal{B}_\mathcal{T}$ denote the Banach space $\mathcal{B}_\mathcal{T}([t_0, T], L_2)$, the family of all $\mathcal{F}_t$-measurable, $\widehat{C}$-valued random variables ψ with the norm

$$\|\psi\|^2_{\mathcal{B}_\mathcal{T}} = \sup_{t\in[t_0,T]} E\|\psi\|^2_t.$$

Let $L_2^0(\Omega, \mathcal{B}_\mathcal{T})$ denote the family of all $\mathcal{F}_0$-measurable, $\mathcal{B}_\mathcal{T}$-valued random variable φ.

Definition 2.1. A semigroup $\{S(t), t \geqslant t_0\}$ is said to be uniformly bounded if $\|S(t)\| \leqslant M$ for all $t \geqslant t_0$, where $M \geqslant 1$ is some constant. If $M = 1$, then the semigroup is said to be contraction semigroup.

Definition 2.2. For a given $T \in (t_0, +\infty)$, a stochastic process $\{x(t) \in \mathcal{B}_\mathcal{T}, -\infty < t \leqslant T\}$ is called a mild solution to system (2.1), (2.2) and (2.3) in $(\Omega, P, \{\mathcal{F}_t\})$, if

(i) $x(t) \in \mathcal{B}_\mathcal{T}$ is $\mathcal{F}_t$-adapted;
(ii) $x(t_0 + s) = \varphi(s)$ when $s \in (-\infty, 0]$, and

$$\begin{aligned} x(t) = \sum_{k=0}^{+\infty}\Bigg[&\prod_{i=1}^{k} b_i(\tau_i)S(t-t_0)[\varphi(0) + g(0,\varphi)] - \prod_{i=1}^{k} b_i(\tau_i)g(t, x_t) \\ &- \left[\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} AS(t-s)g(s, x_s)ds + \int_{\xi_k}^{t} AS(t-s)g(s, x_s)ds\right] \\ &+ \left[\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} S(t-s)f(s, x_s)ds + \int_{\xi_k}^{t} S(t-s)f(s, x_s)ds\right]\Bigg] I_{[\xi_k,\xi_{k+1})}(t), \quad t \in [t_0, T], \end{aligned}$$