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ORIGINAL ARTICLE

Lacunary *I*-convergence in probabilistic *n*-normed space

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Abstract In this article using the concept of ideal and lacunary sequence we introduce the concept of lacunary *I*-convergent, lacunary *I*-Cauchy and lacunary I^*-convergent sequences in probabilistic *n*-normed space.We obtain some results related to these concepts. Also the concept of lacunary refinement of a lacunary sequence is discussed in probabilistic *n*-normed space.

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1. Introduction

The notion of probabilistic metric spaces was introduced by Menger [1]. The idea of Menger was to use distribution function instead of non-negative real numbers as values of the metric. After that it was developed by many authors. Using this concept, Serstnev [2] introduced the concept of probabilistic normed space. Its theory is important as a generalization of deterministic results of linear normed spaces and also in the study of random operator equations. The theory of 2-norm and *n*-norm on a linear space was introduced by Gahler ([3,4]) which was later developed by Tripathy and Borgohain [5], Tripathy and Dutta [6] and many others.

The notion of *I*-convergence was studied at the initial stage by Kostyrko et al. [7]. Later on it was further investigated by Tripathy and Hazarika ([8–11]) Salat et al. [12], Tripathy and Mahanta [13], Tripathy et al. [14] and many others from different aspects.

Now we recall some notations and definitions which will be used in this paper.

Definition 1.1. A probabilistic *n*-normed linear space or in short Pr-n-space is a triplet $(X, \nu, *)$, where X is a real linear space of dimension greater than one, ν is a mapping from X^n into D and $*$, a continuous *t*-norm satisfying the following conditions for every $x_1, x_2, \ldots, x_n \in X$ and $s, t > 0$:

(*i*) $\nu((x_1, x_2, \ldots, x_n), t) = 1$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent.

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(ii) $v((x_1, x_2, \ldots, x_n), t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$.
(iii) $v((x_1, x_2, \ldots, \alpha x_n), t) = v((x_1, x_2, \ldots, x_n), \frac{t}{|\alpha|})$ if $\alpha \neq 0, \alpha \in \mathbb{R}$.
(iv) $v((x_1, x_2, \ldots, x_n + x'_n), s + t) \geqslant v((x_1, x_2, \ldots, x_n), s) * v((x_1, x_2, \ldots, x'_n), t)$.

Example 1.1. Let $(X, ||(\bullet, \bullet, \ldots, \bullet)||)$ be an n-normed linear space. Also let $a * b = \min\{a, b\}$, for $a, b \in [0, 1]$, and $v((x_1, x_2, \ldots, x_n), t) = \frac{t}{t + \|(x_1, x_2, \ldots, x_n)\|}$. Then $(X, v, *)$ is an Pr-n-space.

Definition 1.2. Let $(X, v, *)$ be an Pr-n-space. A sequence $x = \{x_k\}$ in X is said to be convergent to $L \in X$ with respect to the probabilistic n-norm v^n if for every $\varepsilon > 0, \lambda \in (0, 1)$ and $y_1, y_2, \ldots, y_{n-1} \in X$, there exists $k_0 \in \mathbb{N}$ such that $v((y_1, y_2, \ldots, y_{n-1}, x_k - L), \varepsilon) > 1 - \lambda$, for all $k \geqslant k_0$ and we write $v^n - \lim x_k = L$.

Definition 1.3. Let $(X, v, *)$ be a Pr-n-space. A sequence $\{x_k\}$ in X is said to be a Cauchy sequence with respect to the probabilistic n-norm v^n if given $\varepsilon > 0, \lambda \in (0, 1)$ and $y_1, y_2, \ldots, y_{n-1} \in X$, there exists $k_0 \in \mathbb{N}$ such that $v((y_1, y_2, \ldots, y_{n-1}, x_k - x_m), \varepsilon) > 1 - \lambda$, for all $k, m \geqslant k_0$.

Definition 1.4. Let X be a non-empty set. A non-void class $I \subseteq 2^X$ (power set of X) is called an ideal if I is additive (i.e. $A, B \in I \Rightarrow A \cup B \in I$) and hereditary (i.e. $A \in I$ and $B \subseteq A \Rightarrow B \in I$).

Definition 1.5. A non-empty family of sets $\mathfrak{I} \subset 2^X$ is said to be a filter on X if and only if $\emptyset \notin \mathfrak{I}$, for each $A, B \in \mathfrak{I}$, we have $A \cap B \in \mathfrak{I}$ and for each $A \in \mathfrak{I}$ and $B \supset A, B \in \mathfrak{I}$.

For each ideal I there is a filter $\mathfrak{I}(I)$ corresponding to I, given by

$\mathfrak{I}(I) = \{K \subseteq \mathbb{N} : \mathbb{N} \setminus K \in I\}$.

An ideal I is called non-trivial if $I \neq \emptyset$ and $X \notin I$. A non-trivial ideal I is said to be an admissible ideal if it contains all singleton sets.

The usual convergence is a particular case of I-convergence. In this case $I = I_f$ (the ideal of all finite subsets of $\mathbb{N}$).

Definition 1.6. By a lacunary sequence we mean an increasing integer sequence $\theta = (k_r), r = 0, 1, 2, \ldots$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ will be abbreviated by q_r.

The notion of lacunary sequence spaces has been investigated from different aspects by Tripathy and Baruah [15], Tripathy and Dutta [16], Tripathy and Mahanta [17] and many others in the recent years from different aspects.

2. Lacunary I-convergence in Pr-n-space

Definition 2.1. Let $(X, v, *)$ be a Pr-n-space and $\theta = (k_r)$ be a lacunary sequence. A sequence $x = \{x_i\}$ in X is said to be lacunary convergent to $L \in X$ with respect to the probabilistic n-norm v^n if for every $\varepsilon > 0$ and $\lambda \in (0, 1), y_1, y_2, \ldots, y_{n-1} \in X$, there exists $r_0 \in N$ such that

$$\frac{1}{h_r} \sum_{i \in I_r} v((y_1, y_2, \ldots, y_{n-1}, x_i - L), \varepsilon) > 1 - \lambda,$$

for all $r \geqslant r_0$ and we write $(v^n)^\theta - \lim x_k = \text{Ł}$.

Definition 2.2. Let $(X, v, *)$ be a Pr-n-space and $\theta = (k_r)$ be a lacunary sequence. A sequence $x = \{x_i\}$ in X is said to be lacunary I-convergent to $L \in X$ with respect to the probabilistic n-norm v^n if for every $\varepsilon > 0, \lambda \in (0, 1)$ and $y_1, y_2, \ldots, y_{n-1} \in X$, the set

$$\left\{ r \in \mathbb{N} : \frac{1}{h_r} \sum_{i \in I_r} v((y_1, y_2, \ldots, y_{n-1}, x_i - L), \varepsilon) \leqslant 1 - \lambda \right\} \in I$$

and we write $I^\theta_{v(n)} - \lim x_k = \text{Ł}$.

Theorem 2.1. *Let $(X, v, *)$ be a Pr-n-space and θ be a fixed lacunary sequence. If a sequence $x = \{x_i\}$ is lacunary I-convergent with respect to the probabilistic n-norm v^n, then $I^\theta_{v(n)}$-limit is unique.*

Proof. Let us assume that $I^\theta_{v(n)} - \lim x_k = L_1$ and $I^\theta_{v(n)} - \lim x_k = L_2$.

For a given $\lambda > 0$, choose $\eta \in (0, 1)$ such that $(1 - \eta) * (1 - \eta) > 1 - \lambda$. Then for any $\varepsilon > 0$, we define the following sets:

$$K_{v,1}(\eta, \varepsilon) = \left\{ r \in \mathbb{N} : \frac{1}{h_r} \sum_{i \in I_r} v((y_1, y_2, \ldots, y_{n-1}, x_i - L_1), \varepsilon) > 1 - \eta \right\}$$

and $K_{v,2}(\eta, \varepsilon) = \left\{ r \in \mathbb{N} : \frac{1}{h_r} \sum_{i \in I_r} v((y_1, y_2, \ldots, y_{n-1}, x_i - L_2), \varepsilon) > 1 - \eta \right\}$.

Since $I^\theta_{v(n)} - \lim x_k = L_1$, so $K_{v,1}(\eta, \varepsilon) \in \mathfrak{I}(I)$, for all $\varepsilon > 0$.

Also $I^\theta_{v(n)} - \lim x_k = L_2$ gives $K_{v,2}(\eta, \varepsilon) \in \mathfrak{I}(I)$, for all $\varepsilon > 0$.

Now let $K_v(\eta, \varepsilon) = K_{v,1}(\eta, \varepsilon) \cap K_{v,2}(\eta, \varepsilon)$. Then $K_v(\eta, \varepsilon) \in \mathfrak{I}(I)$.

Now if $r \in K_v(\eta, \varepsilon)$, then we have

$$\begin{aligned}
&\frac{1}{h_r} \sum_{i \in I_r} v((y_1, y_2, \ldots, y_{n-1}, L_1 - L_2), \varepsilon) \\
&\quad \geqslant \frac{1}{h_r} \sum_{i \in I_r} v\left((y_1, y_2, \ldots, y_{n-1}, x_i - L_1), \frac{\varepsilon}{2}\right) \\
&\qquad * \frac{1}{h_r} \sum_{i \in I_r} v\left((y_1, y_2, \ldots, y_{n-1}, x_i - L_2), \frac{\varepsilon}{2}\right) \\
&\quad > (1 - \eta) * (1 - \eta) > 1 - \lambda.
\end{aligned}$$

Since $\lambda > 0$ is arbitrary, we have

$$\frac{1}{h_r} \sum_{i \in I_r} v((y_1, y_2, \ldots, y_{n-1}, L_1 - L_2), \varepsilon) = 1,$$

for all $\varepsilon > 0$, which gives $L_1 = L_2$. Therefore $I^\theta_{v(n)}$-limit of (x_n) is unique. □

Theorem 2.2. *Let $(X, v, *)$ be a Pr-n-space, θ be a lacunary sequence and $x = \{x_i\}, y = \{y_i\}$ be two sequences in X. Then*

(i) *If $I^\theta_{v(n)} - \lim x_k = L$ and $\alpha \in \mathbb{R}$, then $I^\theta_{v(n)} - \lim \alpha x_k = \alpha L$.*