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ORIGINAL ARTICLE

Sufficiency and duality in nondifferentiable minimax fractional programming with (H_p, r)-invexity

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Abstract In the present paper, we discuss the optimality condition for an optimal solution to the problem and a dual model is formulated for a non differentiable minimax fractional programming problem. Weak, strong and strict converse duality results are concerned involving (H_p, r)-invexity.

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1. Introduction

Fractional programming is an fascinating and interesting topic for research that appeared in several types of optimization problems. These programming are widely used in different branches of engineering and sciences, for example it can be used in engineering and economics to minimize a ratio of functions between a given period of time and utilized resource in order to measure the efficiency or productivity of a system. In these types of problems the objective function is usually given as a ratio of functions in fractional programming form (see Stancu-Minasian [1]).

Optimization problems with minimax type functions are arise in the design of electronic circuits, however, minimax fractional problems appear in formulation of discrete and continuous rational approximation problem with respect to the Chebyshev norm [2], in continuous rational games [3], in multiobjective programming [4], in engineering design as well as in some portfolio solution problems discussed by Bajaona-Xandari and Martinez-Legaz [5].

Yadav and Mukherjee [6] formulated two dual models for primal problem and derived duality theorem for convex differentiable minimax fractional programming, a step forward Chandra and Kumar [7] improved the dual formulation of Yadav and Mukherjee and they provided two modified dual problems for minimax fractional programming and proved duality results. Lai et al. [8] proved necessary and sufficient optimality conditions for nondifferentiable minimax fractional problem with generalized convexity and applied these optimality conditions to established a parametric dual model and also discussed duality results. Many papers are appeared in this direction (see Yuan et al. [9],

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Ahmad [10,11], Lai et al. [8], Hu et al. [12] and Lai and Lee [13]).

In the course of generalization of convex functions, Avriel [14] first introduced the definition of r-convex functions and established some characterizations and relations between r-convexity and other generalization of convexity . Antczak [16] introduced the concept of a class of r-preinvex functions, which is a generalization of r-convex function and preinvex function, and obtained some optimality results under r-preinvexity. Lee and Ho [15] established necessary and sufficient conditions for efficiency of multiobjective fractional programming problems involving r-invex functions, they also discussed Wolfe and Mond–Weir duality in this setting, Antczak [16] introduced p-invex sets and (p,r)-invex functions as a generalization of invex and preinvex functions. Ahmad et al. [10] worked out the duality in nondifferentiable minimax fractional programming with $B-(p,r)$-invexity. Recently, Jayswal et al. [17] investigated the duality for semi infinite programming problems involving (H_p,r)-invexity. Motivated by Jyaswal et al. [17] and Ahmad et al. [10], in this paper we investigate the duality for minimax fractional programming involving (H_p,r)-invexity.

We consider the following nondifferentiable minimax fractional programming problem

$$\text{Minimize} \quad \psi(x) = \sup_{y\in Y} \frac{f(x,y) + (x^TCx)^{\frac{1}{2}}}{g(x,y) - (x^TDx)^{\frac{1}{2}}} \quad \text{(NFP)}$$

$$\text{Subject to} \quad h(x) \leqslant 0, \quad x \in R^n,$$

where Y is a compact subset of R^l,$f(.,.)$, $g(.,.) : R^n \times R^l \to R$, $h(.,.) : R^n \to R^m$ are C^1 functions. C and D are $n\times n$ positive semidefinite symmetric matrices. Throughout this paper, we assume that $g(x,y) - (x^TDx)^{\frac{1}{2}} > 0$ and $f(x,y) + (x^TCx)^{\frac{1}{2}} \geqslant 0$, for all $(x,y) \in R^n \times R^l$.

2. Preliminaries

We start this section with the following some definitions

Definition 2.1. [18]. The weighted r-mean of a_1 and a_2 $(a_1, a_2 > 0)$ is given by

$$M_r(a_1,a_2;\lambda) = \begin{cases} \left(\lambda a_1^r + (1-\lambda)a_2^r\right)^{\frac{1}{r}} & \text{for } r\neq 0, \\ a_1^{\lambda}a_2^{1-\lambda} & \text{for } r = 0, \end{cases}$$

where $\lambda \in (0,1)$ and $r \in R$.

Definition 2.2. A subset $X \subseteq R^n$ is said to be $H_p - invex$ set, if for any $x,\ u \in X$, there exists a vector function $H_p : X \times X \times [0,1] \to R^n$, such that

$$H_p(x,u;0) = e^u, H_p(x,u;\lambda) \in R^n_+,$$

$$lnH_p(x,u;\lambda) \in X, \quad \forall \lambda \in [0,1], \quad p \in R.$$

Note 2.1. It is understood that the logarithm and the exponentials appearing in the above definition are taken to be component wise.

Throughout the paper, we take X to be a H_p-invex set unless otherwise specified, H_p-right differentiable at 0 with respect to the variable λ for each given pair $x,\ u \in X$ and $f : X \to R$ is differentiable function on X. The symbol $H'_p(x,u;0+) = \left(H'_{p^1}(x,u;0+),\ldots,H'_{p^n}(x,u;0+)\right)^T$ denotes the right derivative of H_p at 0 with respect to the variable λ for each given pair $x,\ u \in X$, $\nabla f(x) = (\nabla_1 f(x),\ldots,\nabla_n f(x))^T$ denotes the differential of f at x, and so $\frac{\nabla f(u)}{e^u} = \left(\frac{\nabla_1 f(u)}{e_1^u},\ldots\frac{\nabla_n f(u)}{e_n^u}\right)^T$.

Note 2.2. All the theorems in the subsequent parts of this paper will be proved only in the case when $r\neq 0$ and $r>0$ (in the case when $r<0$, the direction of some of the inequalities in the proof of the theorems should be changed to the opposite one).

Definition 2.3. A differentiable function $f : X \to R$ is said to be (strictly) (H_p,r)-invex at $u \in X$, if for all $x \in X$, one of the relations

$$\frac{1}{r}[e^{r(f(x)-f(u))} - 1] \geqslant \frac{\nabla f(u)^T}{e^u} H'_p(x,u;0+)(>) \quad \text{for } r\neq 0,$$

$$f(x) - f(u) \geqslant \frac{\nabla f(u)^T}{e^u} H'_p(x,u;0+)\ (>) \quad \text{for} \quad r = 0,$$

hold.

If the above inequalities are satisfied at any point $u \in X$ then f is said to be (H_p,r)-invex (strictly (H_p,r)-invex) on X.

Now we define the generalized (H_p,r)-invex functions as follows.

Definition 2.4. A differentiable function $f : X \to R$ is said to be (H_p,r)-pseudo invex at $u \in X$, if for all $x \in X$, the relations

$$\frac{\nabla f(u)^T}{e^u} H'_p(x,u;0+) \geqslant 0 \quad \Rightarrow \quad \frac{1}{r}[e^{r(f(x)-f(u))} - 1] \geqslant 0, \quad \text{for} \quad r\neq 0,$$

$$\frac{\nabla f(u)^T}{e^u} H'_p(x,u;0+) \geqslant 0 \quad \Rightarrow \quad f(x) - f(u) \geqslant 0, \quad \text{for} \quad r = 0,$$

hold.

Definition 2.4. A differentiable function $f : X \to R$ is said to be (Strictly) (H_p,r)-quasi invex at $u \in X$, if for all $x \in X$, the relations

$$\frac{1}{r}[e^{r(f(x)-f(u))} - 1] \leqslant 0, \quad \Rightarrow \quad \frac{\nabla f(u)^T}{e^u} H'_p(x,u;0+)\ (<) \leqslant 0 \quad \text{for} \quad r\neq 0,$$

$$f(x) - f(u) \leqslant 0, \quad \Rightarrow \quad \frac{\nabla f(u)^T}{e^u} H'_p(x,u;0+)\ (<) \leqslant 0 \quad \text{for} \quad r = 0,$$

hold.

3. Notations and preliminaries

Let $S = \{x \in R^n : h(x) \leqslant 0\}$ denotes the set of all feasible solutions (NFP). An point $x \in S$ is called the feasible point of (NFP). For each $(x,y) \in R^n \times R^l$, we define