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ORIGINAL ARTICLE

Approximating common random fixed point for two finite families of asymptotically nonexpansive random mappings

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Abstract The aim of this paper is to study weak and strong convergence of an implicit random iterative process with errors to a common random fixed point of two finite families of asymptotically nonexpansive random mappings in a uniformly convex separable Banach space.

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1. Introduction

Random approximations and random fixed point theorems are stochastic generalizations of classical approximations and fixed point theorems. The study of random fixed point theorems was initiated by Prague school of probabilities in the 1950s by Spacek [1] and Hans [2,3]. The interest in these problems was enhanced after the publication of the survey article of Bharucha-Reid [4] in 1976. Random fixed point theory and applications have been further developed rapidly in recent years (see e.g. [5–12] and references therein).

The class of asymptotically nonexpansive self-mappings introduced by Goebel and Kirk [13] in 1972. In 2001, Xu and Ori [14] introduced the following implicit iteration process $\{x_n\}$ defined by

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_{n(modN)}x_n, n \geqslant 1, \quad x_0 \in K, \tag{1.1}$$

for a finite family of nonexpansive mappings $\{T_1, T_2, \ldots, T_N\}$: $K \to K$, where K is a nonempty closed convex subset of a Hilbert space E and $\{\alpha_n\}_{n\geqslant 1}$ is a real sequence in (0, 1). They proved the weakly convergence of the sequence $\{x_n\}$ defined by (1.1) to a common fixed point $p \in F = \cap_{i=1}^{N} F(T_i)$.

In 2003, Sun [15] introduced the following implicit iteration process $\{x_n\}$ defined by

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_{i(n)}^{k(n)}x_n, n \geqslant 1, \quad x_0 \in K, \tag{1.2}$$

for a finite family of asymptotically quasi-nonexpansive self-mappings on a bounded closed convex subset K of a Hilbert space E with $\{\alpha_n\}_{n\geqslant 1}$ a sequence in (0, 1), where $n = (k(n)-1)N + i(n)$, $i(n) \in \{1, 2, \ldots, N\}$, and proved the

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strong convergence of the sequence $\{x_n\}$ defined by (1.2) to a common fixed point $p \in F = \cap_{i=1}^{N} F(T_i)$.

In 2010, Filomena Cianciaruso et al. [16] considered the following implicit iterative process for a finite family of asymptotically nonexpansive mappings

$$\begin{aligned} x_n &= (1-\alpha_n-\gamma_n)x_{n-1} + \alpha_n T_{i(n)}^{k(n)} y_n + \gamma_n u_n, \\ y_n &= (1-\beta_n-\delta_n)x_n + \beta_n T_{i(n)}^{k(n)} x_n + \delta_n v_n, \quad n \geqslant 1, \end{aligned} \tag{1.3}$$

where $n = (k(n)-1)N + i(n)$, $i(n) \in \{1, 2, \ldots, N\}$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_n\}$ are sequences of real numbers in (0, 1) with $\alpha_n + \gamma_n \leqslant 1$ and $\beta_n + \delta_n \leqslant 1$ for all $n \geqslant 1$ and $\{u_n\}$, $\{v_n\}$ are two bounded sequences and x_0 is a given point. They proved convergence of the implicit iterative process defined by (1.3) to a common fixed point of asymptotically nonexpansive mappings in uniformly convex Banach spaces.

Very recently, Hao et al. [17] studied the convergence of an implicit iterative process with errors for two finite families $\{T_i\}_{i=1}^{N}, \{S_i\}_{i=1}^{N} : K \to K$ of asymptotically nonexpansive mappings defined as follows:

$$\begin{aligned} x_n &= (1-\alpha_n-\gamma_n)x_{n-1} + \alpha_n T_{i(n)}^{k(n)} y_n + \gamma_n u_n, \\ y_n &= (1-\beta_n-\delta_n)x_n + \beta_n S_{i(n)}^{k(n)} x_n + \delta_n v_n, \quad n \geqslant 1, \end{aligned} \tag{1.4}$$

where $n = (k(n)-1)N + i(n)$, $i(n) \in \{1, 2, \ldots, N\}$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_n\}$, are sequences of real numbers in [0,1] with $\alpha_n + \gamma_n \leqslant 1$ and $\beta_n + \delta_n \leqslant 1$ for all $n \geqslant 1$ and $\{u_n\}$, $\{v_n\}$, are two bounded sequences.

The development of random fixed point iterations was initiated by Choudhury in [18] where random Ishikawa iteration scheme was defined and its strong convergence to a random fixed point in Hilbert spaces was discussed. After that, several authors have worked on random fixed point iterations some of which are noted in ([19–24]) and many others. Banerjee et al. [25] constructed a composite implicit random iterative process with errors for a finite family $\{T_i: i \in I = \{1, 2, \ldots, N\}\}$ of N continuous asymptotically nonexpansive random operators from $\Omega \times C$ to C, where C be nonempty closed convex subset of a separable Banach space E. They discuss the necessary and sufficient conditions for the convergence of this composite implicit random iterative process defined in the compact form as follows:

$$\begin{aligned} &\xi_n(t) = \alpha_n \xi_{n-1}(t) + \beta_n T_{i(n)}^{k(n)}(t, \eta_n(t)) + \gamma_n f_n(t), \\ &\eta_n(t) = a_n \xi_n(t) + b_n T_{i(n)}^{k(n)}(t, \xi_n(t)) + c_n g_n(t), \quad n \geqslant 1, \quad \forall t \in \Omega, \end{aligned} \tag{1.5}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ are sequences of real numbers in [0, 1] with $\alpha_n + \beta_n + \gamma_n = a_n + b_n + c_n = 1$ and $\{f_n(t)\}$, $\{g_n(t)\}$ are bounded sequences of measurable functions from Ω to C.

Inspired and motivated by theses facts, we investigate convergence of the following implicit random iterative process:

Definition 1.1. Let $\{T_i\}_{i=1}^{N}$ and $\{S_i\}_{i=1}^{N}$ be two finite families of 2N asymptotically nonexpansive random mappings form $\Omega \times C$ to C. where C is a nonempty closed convex subset of a separable Banach space E. Let ξ_0: $\Omega \to C$ be a measurable function. Then, define the sequence $\{\xi_n(w)\}$ as

$$\begin{aligned} \xi_n(w) &= (1-\alpha_n-\gamma_n)\xi_{n-1}(w) + \alpha_n T_{i(n)}^{k(n)}(w, \eta_n(w)) + \gamma_n f_n(w), \\ \eta_n(w) &= (1-\beta_n-\delta_n)\xi_n(w) + \beta_n S_{i(n)}^{k(n)}(w, \xi_n(w)) + \delta_n g_n(w), \end{aligned} \tag{1.6}$$

where $n = (k(n)-1)N + i(n)$, $i(n) \in \{1, 2, \ldots, N\}$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\delta_n\}$ are sequences of real numbers in [0,1] with $\alpha_n + \gamma_n \leqslant 1$ and $\beta_n + \delta_n \leqslant 1$ for all $w \in \Omega$ and for all $n \geqslant 1$ and $\{f_n(w)\}$, $\{g_n(w)\}$ are bounded sequences of measurable functions from Ω to C.

We extend the random iterative process (1.5) to the case of two finite families of asymptotically nonexpansive random mappings $\{T_i, S_i: i = 1, 2, \ldots, N\}$ and also study the random version of the implicit iterative process (1.4). We obtain the weak and strong convergence of an implicit random iterative process (1.6) in a uniformly convex Banach space.

2. Preliminaries

Let (Ω, Σ) be a measurable space, C a nonempty subset of E. A mapping ξ: $\Omega \to C$ is called measurable if $\xi^{-1}(B \cap C) \in \Sigma$ for every Borel subset B of a Banach space E. A mapping T: $\Omega \times C \to C$ is said to be random mapping if for each fixed $x \in C$, the mapping $T(.,x)$: $\Omega \to C$ is measurable. A measurable mapping ξ: $\Omega \to C$ is called a random fixed point of the random mapping T: $\Omega \times C \to C$ if $T(w, \xi(w)) = \xi(w)$ for each $w \in \Omega$.

We denote the set of all random fixed points of random mapping T by $RF(T)$.

Definition 2.1 [26]. A Banach space E is said to satisfy the Opial's condition if for any sequence $\{x_n\}$ in E, $x_n \rightharpoonup x$ weakly as $n \to \infty$ and $x \neq y$ implying that

$$\limsup_{n\to\infty} \|x_n - x\| < \limsup_{n\to\infty} \|x_n - y\|,$$

for all $y \in E$.

Definition 2.2. A map T: $C \to E$ is called demiclosed at $y \in E$ if for each sequence $\{x_n\}$ in C and each $x \in E$, $x_n \rightharpoonup x$ weakly and $Tx_n \to y$ strongly imply that $x \in C$ and $Tx = y$.

Definition 2.3 [25]. A finite family $\{T_i: i \in I = \{1, 2, 3, \ldots, N\}\}$ of N continuous random operators from $\Omega \times C$ to E with $F = \bigcap_{i=1}^{N} RF(T_i) \neq \emptyset$ is said to satisfy condition B on C if there exists a nondecreasing function f: $[0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) \geqslant 0$ for all $r \in (0, \infty)$ such that for all $w \in \Omega$, $f(d(\xi(w), F)) \leqslant \max_{1\leqslant i\leqslant N}\{\|\xi(w) - T_i(w, \xi(w))\|\}$ for all $\xi(w)$, where ξ: $\Omega \to C$ is a measurable function and $d(\xi(w), F) = \inf\{\|\xi(w) - q(w)\| : q(w) \in F = \bigcap_{i=1}^{N} RF(T_i)\}$.

Definition 2.4 [19]. Let C be a nonempty closed convex subset of a separable Banach space E and T: $\Omega \times C \to E$ be a random mapping. Then, T is said to be

(1) Nonexpansive random operator if for arbitrary $x, y \in C$,

$$\|T(w,x) - T(w,y)\| \leqslant \|x - y\|, \quad \forall w \in \Omega.$$

(2) Asymptotically nonexpansive random mapping if there exists a measurable mapping sequence $r_n(w)$: $\Omega \to [1, \infty)$ with $\lim_{n\to\infty} r_n(w) = 1$ for each $w \in \Omega$ such that for arbitrary $x, y \in C$ and for each $w \in \Omega$

$$\|T^n(w,x) - T^n(w,y)\| \leqslant r_n(w)\|x - y\|, \quad n = 1, 2, \ldots$$

(3) Uniformly L-Lipschitzian random mapping if there exists a constant $L > 0$ such that for arbitrary $x, y \in C$ and $w \in \Omega$