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ORIGINAL ARTICLE

Approximate boundary controllability of Sobolev-type stochastic differential systems ☆

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Abstract The objective of this paper is to investigate the approximate boundary controllability of Sobolev-type stochastic differential systems in Hilbert spaces. The control function for this system is suitably constructed by using the infinite dimensional controllability operator. Sufficient conditions for approximate boundary controllability of the proposed problem in Hilbert space is established by using contraction mapping principle and stochastic analysis techniques. The obtained results are extended to stochastic differential systems with Poisson jumps. Finally, an example is provided which illustrates the main results.

MATHEMATICS SUBJECT CLASSIFICATION: 34K50; 60H10; 93B05; 93E03

1. Introduction

In many cases, the accurate analysis, design, and assessment of systems subjected to realistic environments must take into account the potential of random loads and randomness in the system properties. Randomness is intrinsic to the mathematical formulation of many phenomena such as fluctuations in the stock market, or noise in communication networks. To build more realistic models in economics, social sciences, chemistry, finance, physics and other areas, stochastic effects need to be taken into account. Mathematical modeling of such systems often leads to differential equations with random parameters. The use of deterministic equations that ignore the randomness of the parameter or replace them by their mean values can result in gross errors. All such problems are mathematically modeled and described by various stochastic systems described by stochastic differential equations, stochastic delay equations, and in some cases stochastic integro-differential equations which are mathematical models for phenomena with irregular fluctuations. Stochastic differential equations are important from the viewpoint of applications since they incorporate (natural) randomness into the mathematical description of phenomena, thereby describing it more accurately. The theory of stochastic differential systems has become an important area of investigation in the past two decades because of their applications to various problems arising in communications, control technology, mechanics,

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electrical engineering, medicine, biology, aviation, spaceflight, material, robot, bioengineering, etc. [1,2]. This is due to the fact that most problems in a real life situation to which mathematical models are applicable are basically stochastic rather than deterministic (see [3]).

Mathematical control theory is one of the important concept in the study of steering the dynamical system from given initial state to any other final state or to neighborhood of the final state under some admissible control input. The controllability problem for an evolution equation is also consist of driving the solution of the system to a prescribed final target state (exactly or in some approximate way) in a finite interval of time (see [4] and references therein). Problems of this type are common in science and engineering and, in particular, they arise often in the context of flow control, the control of flexible structures appearing in flexible robots and large space structures, quantum chemistry, etc. (see [5]). From the mathematical point of view, the problems of exact and approximate controllability are to be distinguished. It is obvious that exact controllability is an essentially stronger notion than approximate controllability. Exact controllability always implies approximate controllability. The converse statement is generally false. However, it should be addressed that in the case of finite dimensional systems, the notions of exact and approximate controllability coincide. Controllability results for a class of fractional-order neutral evolution systems was discussed in [6]. Sakthivel et al, [7] investigated the problem of approximate controllability for a class of nonlinear impulsive differential equations with state-dependent delay by using semigroup theory and fixed point technique. In recent years, controllability problems for various types of deterministic and stochastic dynamical system have been studied in different directions (see [8–13] and references therein). In the literature, there are different definitions of controllability for SDEs, both for linear and nonlinear dynamical systems [8,9,14]. In particular, Klamka [15] derived the stochastic controllability of linear systems with delay in control. Muthukumar et al. [16] proved the approximate controllability of nonlinear stochastic evolution systems with time varying delays with preassigned responses. Sakthivel et al. [17] investigated the approximate controllability of second order stochastic differential equations with impulsive effects by using the Holders inequality, stochastic analysis, and fixed point strategy. Shen et al. [18] proved approximate controllability of abstract stochastic impulsive systems with multiple time-varying delays by using the natural assumptions that the corresponding linear system is approximately controllable. Sakthivel et al. [19,20] studied approximate controllability of fractional stochastic system by using fixed point theorem with stochastic analysis theory.

Especially in the past two decades, applications resulting from technological developments gave rise to the study of infinite dimensional linear systems governed by partial differential equations. In engineering, these systems are referred to as distributed parameter systems. Systems of this type appear for instance in steel making plants, where the heat distribution on a metal slab has to be governed, in biology, where the size of a bacteria population has to be controlled or in electrical engineering, where optimal operation of power plants has to be calculated (see [2]). These examples fit into a class of systems where control cannot be exceeded everywhere. It is for instance only possible to heat the metal slab at the boundary, to control the population size at a certain age or to generate current in the power plants of an electrical network. Several abstract settings have been developed to describe the distributed control systems on a domain in which the control is acted through the boundary. But in these approaches one can encounter the difficulty for the existence of sufficiently regular solution to state space system, the control must be taken in a space of sufficiently smooth functions.

A semigroup approach to boundary input problems for linear differential equations was first presented by Fattorini [21]. This approach was extended by Balakrishnan [22] where he showed that the solution of a parabolic boundary control equation with L_2 controls can be expressed as a mild solution to an operator equation. Barbu [23] investigated a class of boundary distributed linear control systems in Banach spaces. MacCamy et al. [24] obtained the approximate boundary controllability for the heat equations. Han et al. [25] also studied the boundary controllability of differential equations with nonlocal condition by using Banach fixed point theorem. Many authors studied the boundary controllability of differential equations in deterministic cases (see [26–30] and references therein). Balachandran et al. [31] established the sufficient conditions for the boundary controllability of various types of nonlinear Sobolev-type systems including integro differential systems in Banach spaces. A Sobolev-type equation appears in a variety of physical problems such as flow of fluids through fissured rocks, thermodynamics, and propagation of long waves of small amplitude (see [32,33]). Wang [34] addressed the approximate boundary controllability results for semilinear delay differential equations by using the corresponding linear system which is approximately boundary controllable. Li et al. [35] showed that the boundary controllability of nonlinear stochastic differential inclusions by using a fixed point theorem for condensing maps due to Leray-Schauder nonlinear alternative theorem. If the semigroup is compact, then assumptions (H_2) in [35] is valid if and only if the state space is finite dimensional. As a result, the applications are restricted to stochastic ordinary differential control systems. Motivated by [31,34,35], the aim of the proposed work is to obtain the approximate boundary controllability of the following Sobolev-type stochastic differential systems without using the hypothesis (H_2) in [35]

$$\begin{aligned} d(Fx(t)) &= (\rho x(t) + f(t, x(\gamma_1(t)), x(\gamma_2(t)), \ldots, x(\gamma_n(t))))dt \\ &\quad + g(t, x(\gamma_1(t)), x(\gamma_2(t)), \ldots, x(\gamma_n(t)))dW(t), \quad t \in J = [0, b], \\ \tau x(t) &= B_1 u(t), \\ x(0) &= x_0, \end{aligned} \tag{1}$$

where the state variable $x(\cdot)$ takes values in a Hilbert space H with an inner product $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$ and the control function $u(\cdot)$, takes values in Hilbert space U. $B_1 : U \to H$ is a linear continuous operator. Let $\mathcal{C} := C(J; L_2(\Omega, H))$ be the space of all real valued measurable continuous functions from J into H. Let $\rho : D(\rho) \subseteq \mathcal{C} \to R(\rho) \subseteq H$ is a closed, densely defined linear operator, where $D(\rho)$ is the domain of ρ and $R(\rho)$ is the range of ρ and $\tau : D(\tau) \subseteq \mathcal{C} \to R(\tau) \subseteq H$ is a linear operator with τ be a partial differential operator acting on the boundary of H. Let K be a another separable Hilbert space. Suppose $\{W(t)\}_{t \geqslant 0}$ is a given K- valued Brownian motion or Wiener process with a finite trace nuclear covariance operator $Q \geqslant 0$. We are also employing the same notation $\|\cdot\|$ for the norm of $L(K, H)$, where $L(K, H)$ denotes the space of all bounded operators from K into H, simply $L(H)$ if $K = H$. Let $F : D(F) \subset \mathcal{C} \to R(F) \subset H$ be a linear operator, the nonlinear function f be a