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ORIGINAL ARTICLE

On MHD flow of an incompressible viscous fluid

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KEYWORDS

Homotopy Perturbation Method;
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MAPLE 13

Abstract In this paper, we apply Homotopy Perturbation Method (HPM) to find the analytical solutions of nonlinear MHD flow of an incompressible viscous fluid through convergent or divergent channels in presence of a high magnetic field. The flow of an incompressible electrically conducting viscous fluid in convergent or divergent channels under the influence of an externally applied homogeneous magnetic field is studied both analytically and numerically. The graphs are presented to reveal the physical characteristics of flow by changing angles of the channel, Hartmann and Reynolds numbers.

MATHEMATICS SUBJECT CLASSIFICATION: 35K99; 35P05; 35P99

1. Introduction

The incompressible viscous fluid flow through convergent or divergent channels is one of the most applicable cases in many applications such as aerospace, chemical, civil, environmental, mechanical, and biomechanical engineering as well as in understanding rivers and canals. Jeffery [1] and Hamel [2] have carried out the mathematical formulations of this problem in 1915 and 1916, respectively. If we simplify Navier–Stokes equations in the particular case of two-dimensional flow through a channel with inclined walls, finally we can reach Jeffery–Hamel problem [3–6]. Jeffery–Hamel flows have been extensively studied by several authors and discussed in many textbooks, for example [7–11], and so forth. The study of electrically conducting viscous fluid that flows through convergent or divergent channels under the influence of an external magnetic field not only is fascinating theoretically but also finds applications in mathematical modeling of several industrial and biological systems. A possible practical application of the theory we envisage is in the field of industrial metal casting, the control of molten metal flows. Another area in which the theoretical study may be of interest is in the motion of liquid metals or alloys in the cooling systems of advanced nuclear reactors [12]. Clearly, the motion in the region with intersecting walls may represent a local transition between two parallel channels with different cross-sections, a widening or a contraction of the flow. The first recorded use of the word magnetohydrodynamics (MHD) is by Bansal [13]. The theory of MHD is inducing current in a moving conductive fluid in the presence of magnetic field which creates force on electrons of the conductive fluid and also changes the magnetic field itself. A survey of magnetohydrodynamics studies in the mentioned technological field can be found in [14]. The problem is basically an extension of classical Jeffery–Hamel flows of ordinary fluid mechanics to MHD. In the MHD solution an external magnetic field acts as a control parameter for both convergent

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Table 1 Values of α for $Re = 100$ and $\alpha = -2.5°$.

H	0	1000	2000	4000
a	−1.117418863	−0.9644432630	−0.8367326331	−0.6392026162

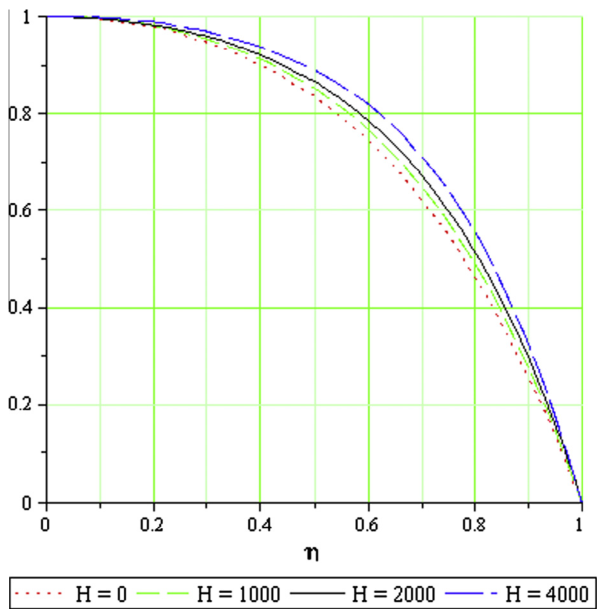

Figure 1 HPM solution for velocity is convergent channel for $Re = 100$ and $\alpha = -2.5°$.

Table 2 Values of α for $Re = 100$ and $\alpha = 2.5°$.

H	0	1000	2000	4000
a	−3.512069452	−3.011764524	−2.583264460	−1.912965835

and divergent channel flows. Here, besides the flow Reynolds number and the channel angular widths, at least an additional dimensionless parameter appears, namely, the Hartman number. Hence, a much larger variety of solutions than in the classical problem are expected. The inspiration of this paper is the extension of a relatively new technique which is called Homotopy Perturbation Method [15–17] to investigate the MHD flow through convergent or divergent channels in presence of a high magnetic field. The governing highly nonlinear equation of this problem is also solved numerically by shooting method, coupled with fourth-order Runge–Kutta scheme.

2. Mathematical formulation

Consider a system of cylindrical polar coordinates (r, θ, z), where the steady two-dimensional flow of an incompressible conducting viscous fluid from a source or sink at channel walls lie in planes and intersect in z-axis. The schematic diagram of problem is illustrated in [18]. Now we assumed that $u_\theta = 0$; it means that there are no changes with respect to z direction; thus the motion is purely in radial direction and merely depends on r and θ and there is no magnetic field along z-axis. The polar form of equation of continuity, Navier–Stokes and Maxwell's in reduce form is given as follows:

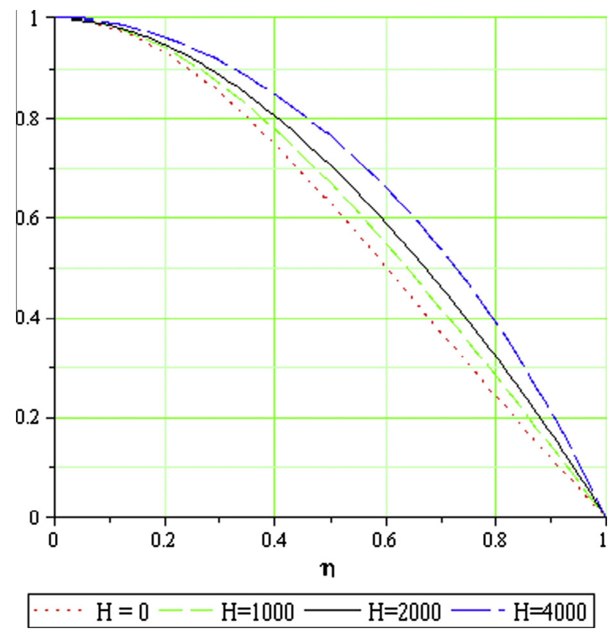

Figure 2 HPM solution for velocity is convergent channel for $Re = 100$ and $\alpha = 2.5°$.

$$\rho\frac{\partial}{r\partial r}[ru(r,\theta)] = 0, \tag{1}$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + v\left[\frac{\partial^2 u(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial\theta^2} - \frac{u(r,\theta)}{r^2}\right] - \frac{\sigma B_0^2}{\rho r^2}u(r,\theta), \tag{2}$$

$$\frac{1}{\rho r}\frac{\partial P}{\partial\theta} - \frac{2v}{r^2}\frac{\partial u(r,\theta)}{\partial\theta} = 0, \tag{3}$$

where B_0 is the electromagnetic induction strength, σ the conductivity of the fluid, u the velocity along radial direction, P the fluid pressure, v the coefficient of kinematic viscosity, and ρ the fluid density.

Now from Eq. (1), we have

$$f(\theta) = ru(r,\theta). \tag{4}$$

Using $\eta = \frac{\theta}{\alpha}$, i.e., the dimensionless parameters, where α is the semiangle between the inclined walls

$$f(\eta) = \frac{f(\theta)}{f_{\max}}, \tag{5}$$

substituting Eq. (5) into Eq. (2) and Eq. (3), we have

$$f'''(\eta) + 2\alpha Ref(\eta)f'(\eta) + (4 - H)\alpha^2 f'(\eta) = 0, \tag{6}$$

where $H = \sqrt{\frac{\sigma B_0^2}{\rho v}}$ is Hartmann number and Re is the Reynolds number is

$$Re = \frac{f_{\max}\alpha}{v}\begin{cases}\text{divergent} - \text{channel}: \alpha > 0, & f_{\max} > 0,\\ \text{convergent} - \text{channel}: \alpha < 0, & f_{\max} < 0.\end{cases}$$

So we have the BCs

$$f(0) = 1, \quad f'(0) = 0, \quad f(1) = 0. \tag{7}$$