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ORIGINAL ARTICLE

An alternative optimization technique for interval objective constrained optimization problems via multiobjective programming

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Abstract An alternative optimization technique via multiobjective programming for constrained optimization problems with interval-valued objectives has been proposed. Reduction of interval objective functions to those of noninterval (crisp) one is the main ingredient of the proposed technique. At first, the significance of interval-valued objective functions along with the meaning of interval-valued solutions of the proposed problem has been explained graphically. Generally, the proposed problems have infinitely many compromise solutions. The objective is to obtain one of such solutions with higher accuracy and lower computational effort. Adequate number of numerical examples has been solved in support of this technique.

MATHEMATICAL SUBJECT CLASSIFICATION: 90C29; 90C30; 90C59; 90C90

1. Introduction

In the context of present day socio-economic scenario, the uncertainty handling optimization techniques are most powerful to increase the productivity of business companies and public organizations. The existence of impreciseness is inevitable in real world data most of which are collected from some insufficient information. While formulating mathematical models, the impreciseness may also come into the existence due to decision-making under uncertain situations. At present, it is a burning question to the researchers: How to model this impreciseness properly to handle the complicated uncertain situations arisen in reality and also how to develop the appropriate solution methodologies? Stochastic [1–3], fuzzy [4–6], or grey optimization techniques [7,8] are some conventional and very familiar approaches to tackle these problems. Each of these methods has some advantages and shortcomings. Alternatively, to deal with the ambiguity of the available data or the impreciseness of any parameter, one may replace those by intervals. An interval can bound the uncertainty/impreciseness

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within its upper and lower limits. Sengupta and Pal [9] have explained the advantages of using intervals to represent uncertain or imprecise parameters over fuzzy set theoretic or probabilistic approaches for solving real world decision-making problems. The main privilege of using interval-oriented techniques is that one has to calculate only the bounds of the intervals which specify the limits of uncertainty. By using intervals and interval-oriented techniques, one can handle the uncertainty/imprecision in a deterministic way [10]. Several researchers [11–17] have used intervals successfully to represent impreciseness and also modeled many real world application problems in interval form. According to Ishibuchi and Tanaka [18], if the imprecise data are represented by intervals, then the expected value of the data can be specified by the centers of the intervals and the uncertainty can be measured by the widths. However, in most of the interval-oriented techniques, there arise some important questions regarding the ranking of arbitrary interval numbers during the implementation. Sometimes, it becomes the key factor to measure the efficiency of the technique. Regarding interval ranking, a pioneering work has been done by Moore [19]. After Moore [19], a number of interval ordering definitions [11,18,20–22] have been developed in different ways to serve various purposes. Detailed survey of these ranking definitions has been given in [9,23] with their advantages and shortcomings. The primary goal of these definitions is to develop reliable solution technique for interval optimization problems with the help of interval ranking.

The primary developments of the concept of interval numbers and their analytical characteristics along with the applications of different branches of mathematics have been provided by Moore [19]. Recently, Moore et al. [24] have given an extensive version of their previous works with the application of INTLAB software in interval analysis. There exist various approaches to solve interval-oriented optimization problems. Some of these approaches ensure the guarantee to enclose the set of all optimal solutions covering all possibilities [25–28]. In the second approach, the aim is to give some approximations of compromise solution [9,18,29,30]. Many of the optimization techniques are developed on the basis of Branch and Bound (*B&B*) algorithms. On the other hand, several simple prototype algorithms for noninterval constrained/unconstrained optimization problems were given by [19,31–33]. Jaulin et al. [12] and Kearfott [13] have provided an illustrative overview of the state-of-the-art of rigorous interval analysis with its applications in optimization problems for global optimality. However, most of the interval-oriented algorithms have been applied to solve noninterval-valued optimization problems. Ratschek and Rokne [34,35] have given some valuable discussions about the interval tools for global optimality including the accelerating devices (i.e., by modifying the algorithm) for rapid convergence. Previously, many researchers developed different types of interval-oriented algorithms/optimization techniques [16,18,20,36–39] for interval linear systems. Ishibuchi and Tanaka [18] proposed a method for linear optimization problems with interval objective functions by converting those into multiobjective optimization problem. An interval-oriented approach of obtaining rigorous solution of linear programming problems with uncertain data has been given in [25]. The solution set, in this case, defines very sharp and guaranteed error bounds and also the method permits a rigorous sensitivity analysis. Chanas and Kuchta [20] have generalized the works of [18] with the help of t_0, t_1-cut of the intervals and developed the general method using multiobjective programming for interval linear programming. A different technique for interval linear optimization problems with interval objective function was proposed by Inuiguchi and Sakawa [36] by introducing the minimax regret criterion. The repeated use of the well known simplex method is the basis of this method from a starting reference solution set. Another approach by using an efficient interval ordering (Acceptability index method [11]) for an interval linear programming problem (ILPP) has been given in [9]. Some previous developments in the solution methodology of ILPP have been given by Fiedler et al. [40]. Recently, Hladík [27] and Gabrel et al. [41] have introduced two different methods for interval linear programming problems. Suprajitno and Bin Mohd [42] have used the modified simplex method for interval linear programming problems. An optimization technique has been proposed by Allahdadi and Nehi [43] to determine the optimal solution set of the ILPP by using the best and worst case (BWC) methods. Hladik [44] proposed a novel algorithm for testing basis stability for ILP. Besides these, the survey work of Hladík [45] contains detailed discussions of the state-of-the-art for the recent developments of ILPP.

However, most of these techniques are restricted only to linear programming problems with inequality constraints. Consideration of nonlinearity in the structure of model formulation is inevitable for most of the engineering, financial or managerial decision-making problems. Liu and Wang [26] have investigated the solution methodology for Quadratic programming problems (QPP) with interval coefficients. In this case, the problem is transformed into a pair of two-level programming problems and applying the duality theorem and the variable transformation technique, the pair of two-level mathematical programming problem is transformed to the conventional one-level QPP. Recently, Jiang et al. [30] prescribed an optimization technique for nonlinear programming problems with interval coefficients by using genetic algorithm (GA) and multiobjective optimization technique. Hladík [28] has proposed a technique to determine the optimal bounds for nonlinear programming problems with interval data that ensures the exact bounds to enclose the set of all optimal solutions. Bhurjee and Panda [46] have introduced a technique for general interval optimization problems. The interval-valued problem is transformed into interval free problem for finding the efficient solutions of the original problem. Parametric representation of interval-valued functions and its important analytic properties are studied and it is used to the newly developed optimization technique.

It is already stated that most of the techniques developed for solving classical/interval-valued constrained/bound-constrained/unconstrained optimization problems are based on Branch and Bound (*B&B*) algorithm which consists of the following four steps: (i) branching of the prescribed search region, (ii) bounding of interval objective values, (iii) comparison of a continuum of interval values, and (iv) choosing of an optimum value. Two different multisplitting techniques for global solution of nonlinear bound-constrained optimization problems have been introduced by Karmakar et al. [47] and they suggested that the multisection division technique is more