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ORIGINAL ARTICLE

Lie symmetry group analysis of magnetic field effects on free convective flow of a nanofluid over a semi-infinite stretching sheet

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Abstract We investigate the steady two-dimensional flow of an incompressible water based nanofluid over a linearly semi-infinite stretching sheet in the presence of magnetic field numerically. The basic boundary layer equations for momentum and heat transfer are non-linear partial differential equations. Lie symmetry group transformations are used to convert the boundary layer equations into non-linear ordinary differential equations. The dimensionless governing equations for this investigation are solved numerically using Nachtsheim–Swigert shooting iteration technique together with fourth order Runge–Kutta integration scheme. Effects of the nanoparticle volume fraction ϕ, magnetic parameter M, Prandtl number Pr on the velocity and the temperature profiles are presented graphically and examined for different metallic and non-metallic nanoparticles. The skin friction coefficient and the local Nusselt number are also discussed for different nanoparticles.

MATHEMATICS SUBJECT CLASSIFICATION: 76D; 76D10; 76W; 80A; 82D80

1. Introduction

Nanotechnology is considered as a significant factor effecting the next major industrial revolution of the current century. Many researchers over the past decade have focused on modeling the thermal conductivity and examined different viscosities of nanofluid because the thermal conductivity of these fluids play an important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Therefore, numerous methods have been taken to improve the thermal conductivity of fluid by suspending nano/micro sized particles/materials in basic fluids such as oil, water and ethylene glycol mixture which are poor heat transfer fluids [1–4].

It is worth mentioning that the boundary layer flow of nanofluids have been recently considered by several authors. Khan and Pop [5] analyzed the development of steady

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Nomenclature

B_0	magnetic field strength	σ	electric conductivity
M	magnetic parameter	ϕ	the solid volume fraction
T	local temperature of the fluid	ρ_{nf}	the effective density of the nanofluid
Pr	Prandtl number	ρ_f	density of the pure fluid
$Re_x^{1/2}C_f$	reduced skin friction coefficient	ρ_s	density of the nanoparticles
$Re_x^{-1/2}Nu_x$	reduced Nusselt number	μ_{nf}	effective dynamic viscosity of the nanofluid
k_{nf}	thermal conductivity of the nanofluid	μ_f	dynamic viscosity of the basic fluid
k_f	thermal conductivity of the base fluid	η	space variable
k_s	thermal conductivity of the nanoparticles	ψ	stream function
$\bar{u}, \bar{v}$	velocity component in $\bar{x}$ and $\bar{y}$ direction	α_{nf}	thermal diffusivity of the nanofluid
$\bar{x}, \bar{y}$	coordinates along and perpendicular to the sheet		

boundary layer flow, heat transfer and nanoparticle volume fraction over a linear stretching surface in a nanofluid. Ahmed and Pop [6] studied the steady mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids using different types of nanoparticles such as Cu, Ag, Al_2O_3 and TiO_2. Kuznetsov and Nield [7] investigated the classical problem of free convection boundary layer flow of viscous and incompressible fluids past a vertical flat plate to the case of nanofluids. In recent years several numerical studies on the modeling of heat transfer in nanofluids have been published by many authors [8,9].

The study of magnetic field effects has important applications in physics, chemistry and engineering. Many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. Mention may be made of drawing, annealing, and thinning of copper wires. In all these cases, the properties of the final product depend to a great extent on the rate of cooling by drawing such strips in an electrically conducting fluid subject to a magnetic field and the characteristic desired in the final product. In view of these applications many authors have investigated the magnetic field effect on ordinary and visco-elastic fluids. Very recently, Hamad [10] studied the effect of magnetic field over a stretching sheet in nanofluids analytically. Keeping this in mind we obtained a numerical solution for the convective heat transfer of an incompressible viscous nanofluid flow over a semi-infinite stretching sheet in the presence of magnetic field effect using lie symmetry group transformation.

2. Formulation of the problem

Consider the steady laminar two-dimensional flow of an incompressible viscous nanofluid over a linearly semi-infinite stretching sheet. We also consider influence of a constant magnetic field of strength B_0 which is applied normally to the sheet (see Figure 1). The temperature at the stretching surface takes the constant value T_w, while the ambient value, attained as y tends to infinity, takes the constant value T_∞. The fluid is a water based nanofluid containing different types of nanoparticles: Copper (Cu), Alumina (Al_2O_3), Silver (Ag) and Titanium Oxide (TiO_2). It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo physical properties of the nanofluid are considered as given in Table 1 (see Hamad [10]). Under the above assumptions, the boundary layer equations governing the flow and concentration field can be written in the dimensional form as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \left(\frac{\mu_{nf}}{\rho_{nf}}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2 \bar{u}}{\rho_{nf}} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha_{nf}\frac{\partial^2 T}{\partial \bar{y}^2} \tag{3}$$

where $\bar{x}$ is the coordinate along the sheet, $\bar{u}$ is the velocity component in the $\bar{x}$ direction, $\bar{y}$ is the coordinate perpendicular to the sheet, $\bar{v}$ is the velocity component in the $\bar{y}$ direction, B_0 is the constant magnetic field strength, T is the local temperature of the fluid, α_{nf} is the thermal diffusivity of the nanofluid, ρ_{nf} is the effective density of the nanofluid, μ_{nf} is the effective dynamic viscosity of the nanofluid, $(\rho C_p)_{nf}$ is the heat capacitance and k_{nf} is the thermal conductivity of the nanofluid are given as

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{4}$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \tag{5}$$

$$k_{nf} = k_f\left\{\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}\right\} \tag{6}$$

Here, ϕ is the solid volume fraction.

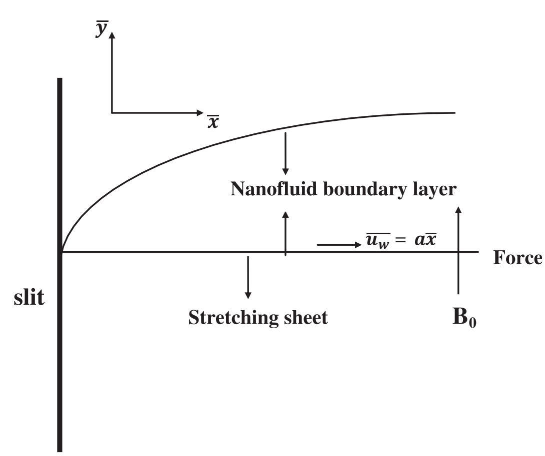

Figure 1 A sketch of the physical model.