Available online at www.sciencedirect.com

ScienceDirect

Karbala International Journal of Modern Science 1 (2015) 1−8
http://www.journals.elsevier.com/karbala-international-journal-of-modern-science/

Reliability extrema of a complex circuit on bi-variate slice classes

Z.A.H. Hassan [a,1], V. Balan [b,*]

[a] Department of Mathematics, College of Education for Pure Sciences, University of Babylon, Babylon, Iraq
[b] Department of Mathematics and Informatics, Faculty of Applied Sciences, University Politehnica of Bucharest, Splaiul Independentei 313, RO-060042, Bucharest, Romania

Received 15 July 2015; revised 8 August 2015; accepted 19 August 2015
Available online 26 September 2015

Abstract

Within the frame of reliability models, the geometry of constant level sets of the reliability function of a complex circuit − regarded as hypersurfaces, reveals properties which provide useful information on the relation between the reliability of the circuit and its components. A special role plays the study of intersections of these hypersurfaces with 2-dimensional plane slices, which provide a foliation by pencils of algebraic curves. The present study classifies these pencils and consequently, it allows: (i) to evaluate the possible bounds of the bivariate slice-reliability in terms of the circuit components; (ii) to compensate the impact of a slice-component malfunction on the slice-reliability, by tuning the appropriate pairing slice-component; (iii) to flag out the cases when the slice-reliability is linear (mono- or bi-variate) in the slice-components, or constant along the whole slice; (iv) in the quadratic case, to make use the convex/concave mutual dependence of slice-components along the curves of constant-slice reliability, in order to maintain or increase the circuit reliability.

MSC2010: 60K10; 62N05; 90B25 90B10; 90B15; 68R10; 94C15; 97K30

Keywords: Reliability; Network model; Algebraic curves; Optimal design

1. Introduction

Numerous studies describe multiple methods to derive the reliability of a complex circuit: the minimal cut method, the sum of disjoint product, the Boolean truth table, the inclusion-exclusion method, etc (see, e.g., [1−23]). Recent research proficiently use geometry tools in studying the reliability function, and prove that the properties of the specific algebraic varieties provide useful information for the study of the modeled complex system ([4−6,12,16]).

In this study, after presenting a brief introduction in reliability of complex systems, we investigate the reliability of a classic bridge complex system, by classifying the pencils of curves of constant reliability lying inside 2-dimensional slices of the foliation given

* Corresponding author.
E-mail addresses: zaher_haddi@yahoo.com, mathzahir@gmail.com (Z.A.H. Hassan), vladimir.balan@upb.ro, vbalan@mathem.pub.ro (V. Balan).
Peer review under responsibility of University of Kerbala.
[1] Currently: Ph.D. student at Department of Mathematics and Informatics, Faculty of Applied Sciences, University Politehnica of Bucharest, Splaiul Independentei 313, RO-060042, Bucharest, Romania.

http://dx.doi.org/10.1016/j.kijoms.2015.08.002

by the constant level reliability hypersurfaces. We use this classification to enhance the design of the system in order to maintain or increase its reliability.

In Section 1 we obtain, by using the path tracing method, the polynomial reliability function of the system, which is further used to derive the constrained reliability function (*slice-reliability*) which depends on two arguments, while the remaining three ones are considered as fixed parameters.

For each choice of selecting the two slice variables, we determine the extremal values of the system slice-reliability in terms of the fixed three parameters, and provide the conditions for which the slice-reliability is quadratic/linear/constant.

As well, for each choice, we show that the slice-reliability may exhibit a *quadratic* (hyperbolic convex or concave) dependence, an *affine* (bi- or mono-variate) dependence, or *constancy* within the slice.

In the *quadratic* case, the pencils of constant slice-reliability curves prove to be mostly sets of hyperbolic-type pencils of conics, parametrized by the arbitrarily admissible reliability level. It is shown that, for constant slice-reliability, the hyperbolic dependence between the free variables provides a center of symmetry with a specific location (0101 and 1010 Cohen-Southerland codes relative to the main clipping domain $[0,1]^2$ of the two active variables), which ensures that a gradual failure of one component may be compensated by improving the pairing component by a technique which uses the convexity/concavity property of the pencil class.

In the *linear* case, the slice-reliability is shown to generally depend on one single variable (the complementary one having an obsolete role), with one singular case exception, when *both the active variables* effectively influence the slice-reliability of the system.

We further illustrate this classification both by pencil representatives of algebraic curves and by the Monge surfaces of the slice-reliability mappings. The classification is then used to describe the dependence of the slice-reliability on the components of the complex system, aiming – by tuning the selected slice-components – to maintain or improve its reliability.

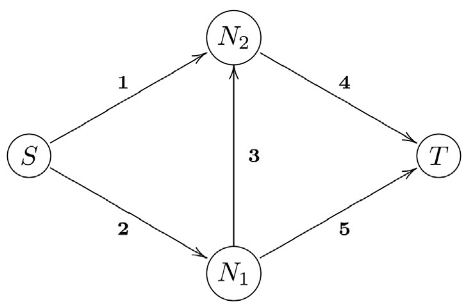

Fig. 1. A bridge-type network.

2. Basic reliability facts

We will briefly describe the basic concepts of network topology and of graph theory [12,13,16,22], commonly used in the network reliability models.

We represent our directed networks as graphs $G = (V,E)$, where V is the set of *vertices* (or *nodes*) and E the set of *edges*, or *arcs*.

In such a configuration (see Fig. 1), one node is considered as the *source* (node S), and a second one is regarded as a *sink*, or *target* (node T). The nodes of the network are joined by numbered arcs.

A *failure of an arc* is equivalent by the failing of the communication along the respective arc (removing, or cutting the arc). The system is *successful* if there exists a valid path which joins the source to the sink. The system is said to *fail* if no such path exists. The *reliability of the system* is the probability that there exist one or more successful paths from the source to the sink.

2.1. Definitions

a) A *path* of the network is a set of arcs, whose success imply the system being successful. For example, the system from Fig. 1 admits as paths the sets {1,4}, {2,5} and {2,3,4}.
b) A *minimal path* (briefly, *min-path*) of the system is a set of arcs which contains a path, such that the removal of any of its arcs produces a subset which no longer contains paths. In other words, if any of the arcs of the minimal path fails, then the system will fail along the remaining arcs of this path. In terms of the network model, the minimal path corresponds to a simple path from the source to the sink in the network.

Considering the set of components of a minimal path $A = \{i_1,\ldots,i_r\}$, and denoting by X_i the indicator for the success of the component a_i, for each $i \in \{i_1,\ldots,i_r\}$, the event of A being successful is the binary function $\Pi_{i=1}^{r} X_i$, and the event of failed path A is $(1 - \Pi_{i=1}^{r} X_i)$, In our case, we have the min-paths

$$A_1 = \{1,4\}, \quad A_2 = \{2,5\}, \quad \text{and} \quad A_3 = \{2,3,4\}.$$