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Medical image denoising using dual tree complex thresholding wavelet transform and Wiener filter

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Abstract Image denoising is the process to remove the noise from the image naturally corrupted by the noise. The wavelet method is one among various methods for recovering infinite dimensional objects like curves, densities, images, etc. The wavelet techniques are very effective to remove the noise because of their ability to capture the energy of a signal in few energy transform values. The wavelet methods are based on shrinking the wavelet coefficients in the wavelet domain. We propose in this paper, a denoising approach basing on dual tree complex wavelet and shrinkage with the Wiener filter technique (where either hard or soft thresholding operators of dual tree complex wavelet transform for the denoising of medical images are used). The results proved that the denoised images using DTCWT (Dual Tree Complex Wavelet Transform) with Wiener filter have a better balance between smoothness and accuracy than the DWT and are less redundant than SWT (StationaryWavelet Transform). We used the SSIM (Structural Similarity Index Measure) along with PSNR (Peak Signal to Noise Ratio) and SSIM map to assess the quality of denoised images.

1. Introduction

The advances of digital imaging technologies include Magnetic Resonance Imaging (MRI), the different digital vascular radiological processes, the cardiovascular and contrast imaging, mammography, diagnostic ultrasound imaging, nuclear medical imaging with Single Photon Emission Computed Tomography (SPECT), Positron Emission Tomography (PET) and multi-detector computed tomography (MDCT). All these processes are producing ever-increasing of images are different from typical photographic images primarily because they reveal internal anatomy as opposed to an image of surface (Rangayyan, 2005), they have revolutionized modern medicine, largely due to technical advances in imaging hardware and new imaging methodologies, the quality of digital medical images becomes an important issue. To achieve the best possible diagnosis it is important that medical images be sharp, clear, and free of noise. Noise removal is essential in medical imaging applications in order to enhance and recover fine details that may be hidden in the data (Satheesh and Prasad, 2011).

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2. Dual tree complex wavelet transform

Complex Wavelet Transforms (CWT) use complex-valued filtering (analytic filter) that decomposes the complex signals into real and imaginary parts in the transform domain. The real and imaginary coefficients are used to compute amplitude and phase information, just the type of information needed to accurately describe the energy localization of oscillating functions (wavelet basis). Another approach to implement an expansive CWT first applies a Hilbert transform to the data. The real wavelet transform is then applied to both the original data and the Hilbert transformed data, and the coefficients of each wavelet transform are combined to obtain a CWT.

The dual-tree complex wavelet transform (DTCWT) is a relatively recent enhancement to the discrete wavelet transform (DWT), with important additional properties: It is nearly shift invariant and directionally selective in two and higher dimensions. It achieves this with a redundancy factor of only 2^{d} for d-dimensional signals, which is substantially lower than the Stationary DWT (Selesnick et al., 2005). Extension of the DTCWT to two dimensions is achieved by separable filtering along columns and then rows. However, if both column and row filters suppress negative frequencies, then only the first quadrant of the 2-D signal spectrum is retained. It is well known, from 2-D Fourier transform theory, that two adjacent quadrants of the spectrum are required to represent fully a real 2-D signal. Therefore, in the DTCWT it is also filtered with complex conjugates of the row (or column) filters in order to retain a second (or fourth) quadrant of the spectrum (Kingsbury, 1999).

The dual tree complex DWT of a signal $x(n)$ is computed using two critically-sampled DWTs in parallel to the same data as shown in the following figure (Fig. 1). If the same filters are used in the upper tree and lower tree nothing is gained. So the filters in this structure will be designed in a specific way that the sub bands of upper DWT are interpreted as real part of complex wavelet transform and the lower tree as imaginary part as shown in Fig. 1. The transform is expansive by factor 2 and shift invariant (Naga Prudhvi Raj and Venkateswarlu, 2012).

3. Wavelet thresholding

Wavelet thresholding is a widely used term for wavelet domain denoising. Denoising by thresholding in wavelet domain has been developed principally by Donoho and Johnstone (1994) and Donoho (1995). In wavelet domain, large coefficients correspond to the signal, and small ones represent mostly noise. The denoised data are obtained by inverse-transforming the suitably thresholded, or shrunk coefficients.

Suppose $s = s_{i,j},\ i = \overline{1,M}$ and $j = \overline{1,N}$ is an image of $M \times N$ pixels, which is corrupted by independent and identically distributed (i.i.d.) zero mean, $n_{i,j}$ are independent standard normal $N(0,1)$ random variables and σ the noise level may be known or unknown. The noise signal can be denoted as $n_{i,j} \sim N(0,\sigma^2)$. This noise may corrupt the signal in a transmission channel. The observed, noise contaminated, image is $x = x_{i,j},\ i = \overline{1,M}$ and $j = \overline{1,N}$.

Therefore, the noised image can be expressed as:

$$x = s + \sigma n_{ij}. \tag{1}$$

The wavelet shrinkage denoising of signal $x(n)$, in order to recover $y(n)$ as an estimate of original signal $s(n)$ is represented as a 4-step algorithm (Taswell, 2000) with j representing decomposition levels, W is forward WT and W^{-1} is inverse WT.

1. $\omega_j = W(x),\ j = 1$ to J.
2. $\lambda_j =$ Level adaptive threshold selection (ω_j).
3. $z_j =$ Thresholding(ω_j, λ_j).
4. $y = W^{-1}(z_j)$.

The standard thresholding of wavelet coefficients is governed mainly by either « *hard* » or « *soft* » thresholding function as shown in Fig. 2. The first function in Fig. 2a is a linear function, which is not useful for denoising, as it does not alter the coefficients. The linear characteristic is presented in the figure just for comparing the non-linearity of other two functions. The hard thresholding function is given as:

$$\begin{cases} z = hard(\omega) = \omega, & |\omega| > \lambda \\ z = hard(\omega) = 0, & |\omega| \leqslant \lambda \end{cases}, \tag{2}$$

where ω and z are the input and output wavelet coefficients respectively. $\lambda$is a threshold value selected.

Similarly, soft thresholding function is given as:

$$\begin{cases} z = soft(\omega) = \operatorname{sgn}(\omega)\max(|\omega| - \lambda, 0), & |\omega| > \lambda \\ z = soft(\omega) = 0, & |\omega| \leqslant \lambda \end{cases}. \tag{3}$$

Thresholding methods can be grouped into two categories, global thresholds and level dependent thresholds. The former method chooses a single value for threshold λ to be applied globally to all empirical wavelet coefficients while the latter method uses different thresholds for different levels. In this work, we have used the universal threshold, which is a simple entropy measure totally dependent on the size of the signal

$$\lambda = \sigma\sqrt{2\log(k)},$$

where k is the size of the signal and λ is the threshold value.These thresholds require an estimate of the noise level σ. The usual standard deviation of the data values is clearly not a good estimator (Ismail and Anjum Khan, 2012; Chang et al., 2000; Donoho, 1995), unless the underlying function S is reasonably flat. Donoho and Jhonstone considered estimating σ in the wavelet domain and suggested a robust estimate that is based only on the empirical wavelet coefficients at the finest resolution level. The reason for considering only the finest level is that the corresponding empirical wavelet coefficients tend to consist mostly of noise. Since there is some signal present even at this level, Donoho and Jhonstone proposed a robust estimate of the noise level σ based on the MAD (Median Absolute Deviation) (Naga Prudhvi Raj and Venkateswarlu, 2012), given by

$$\hat{\sigma}_{(\mathrm{MAD})} = \frac{median\{|x_{i,j}|\}}{0,6745}, \tag{4}$$

where $x_{i,j}$ represents the detail coefficients at the finest level.

4. Wiener filter and noise reduction

Wiener filter was adopted for filtering in the spectral domain. Wiener filter (a type of linear filter) is used for replacing the