Operations Research Perspectives 2 (2015) 73–80

Contents lists available at ScienceDirect

Operations Research Perspectives

journal homepage: www.elsevier.com/locate/orp

# A decision dependent stochastic process model for repairable systems with applications

Paul F. Zantek [a], Timothy Hanson [b,*], Paul Damien [c], Elmira Popova [d]

[a] American Medical Systems, 10700 Bren Rd. W., Minnetonka, MN 55343, United States
[b] University of South Carolina, 1523 Greene St., Columbia, SC 29208, United States
[c] University of Texas at Austin, 1 University Station, B6000, Austin, TX 78712, United States
[d] University of Texas at Austin, 301 E. Dean Keeton St., Austin, TX 78705, United States

ARTICLE INFO

*Article history:*
Available online 30 March 2015

*Keywords:*
Kijima model
Decision dependence
Maintenance schedule
Repairable system
Spiral down effect

ABSTRACT

Management decisions regarding maintenance protocols critically hinge on the underlying probability distribution of the time between failures in most repairable systems. Replacement of the system with a new one resets the system age to zero, whereas a repair does not alter the system age but may shift the *parameters* of the failure-time distribution. Additionally, maintenance decisions lead to left-truncated observations, and right-censored observations. Thus, the underlying stochastic process governing a repairable system evolves based on the management decision taken.

This paper mathematically formalizes the notion of how management actions impact the functioning of a repairable system over time by developing a new stochastic process model for such systems. The proposed model is illustrated using both simulated and real data. The proposed model compares favorably to other models for well-known data on Boeing airplanes. The model is further illustrated and compared to other models on failure time and maintenance data stemming from the South Texas Project nuclear power plant.



## 1. Introduction

Many repairable processes have finite lifetimes that may require *corrective maintenance* (CM) during their lifetimes, such as adjustment, restoration, or lubrication. The process owner could instead opt for replacement with a new process, referred to as *preventive maintenance* (PM); an example of this kind of repair would be a complete overhaul of the system. An important application, considered later in this article, is nuclear-power generation. A nuclear power plant is comprised of numerous systems that fail at random times, thus requiring frequent maintenance. At each maintenance time, management decides whether the maintenance should be corrective or preventive, and this potentially influences the length of time until the next system failure. Failures may have significant implications for safety and operating costs, as well as the ability to satisfy customer demand for electricity.

A widespread assumption in the reliability literature is that the parameters of the *failure-time intensity* are unchanged after CM, commonly termed the *minimal repair assumption* or *minimal repair hypothesis*. In order to test and, if necessary, relax the minimal repair assumption, a new stochastic process is introduced in which the failure intensity following a CM is allowed to be distinctly different than that following a PM; the failure intensity can reflect repairs that improve reliability or make it worse. Relevant properties of this stochastic process are characterized, and a two-stage procedure is proposed for maximum likelihood estimation of its parameters. As a byproduct of the maximum likelihood estimation, Wald confidence intervals for the parameters of the failure-time distribution are constructed. In addition, a likelihood ratio test (LRT) of the minimal repair hypothesis is developed.

A crucial, practical feature in any repairable system is the presence of right censored failure times. Our stochastic process model allows for such events, namely when maintenance is performed prior to a failure occurring, common in maintenance schedules. The properties of our methods, including coverage probabilities of the Wald confidence intervals and the sensitivity of the LRT, are studied using simulation.

To exemplify the methodological advances, we analyze two datasets: (a) a classic repairable systems dataset on Boeing air conditioners, and (b) the maintenance history and failure times for

* Corresponding author.
*E-mail addresses:* Paul.Zantek@ammd.com (P.F. Zantek), hansont@stat.sc.edu (T. Hanson), paul.damien@mccombs.utexas.edu (P. Damien).

http://dx.doi.org/10.1016/j.orp.2015.03.002

a water chilling system in the South Texas Project Nuclear Power Plant.

The stochastic process model considered in this article is related to models of repairable systems in the reliability literature; for comprehensive reviews of this literature, see [1,2], and [3]. Renewal processes are commonly used if all the maintenance repairs are PM, bringing the system to a "good-as-new" state each time (known as *perfect repair*). This assumption simply restarts a common failure intensity to its value at zero after every repair. Note then that the term *good-as-new* is misleading in the presence of decreasing intensity (and inter-failure hazard), as systems that have not been repaired recently are actually more reliable than those that have; this phenomenon is seen in the power plant data in Section 5.

Non-homogeneous Poisson processes (NHPP) are used if all repairs are CM, i.e. bring the system to a "good-as-old" state (known as *minimal repair*), leaving the failure intensity unchanged; this can happen, for example, by replacing a failed sub-component of a system. The NHPP is formally nested in the model proposed in Section 2. Although common, the basic assumption of a consistently "minimal" CM repair is questionable; usually several types of maintenance, with varying degrees of effectiveness, are undertaken throughout the lifetime of the system. For a recent example assuming minimal repair, see [4].

Brown and Proschan [5] assume that repairs are either good-as-new (PM) or bad-as-old (minimal repair) with probabilities $p$ and $1 - p$, respectively. Block, Borges and Savits [6] allow these probabilities $p(t)$ to vary with system age; Whitaker and Samaniego [7] assume the type of maintenance is known. Doyen [8] presents a nonparametric estimation approach to this model for unknown but fixed $p$ along with a review of recent literature on imperfect repairs and maintenance scheduling. Presnell, Hollander and Sethuraman [9] develop a test for the minimal repair assumption in a particular model that Block, Borges and Savits [6] proposed; however, if minimal repair is rejected, the question remains as to whether CM makes the system better or worse than in the case of minimal repair. In many applications this distinction is crucial. If one ignores maintenance decisions, Cooper, de Mello and Kleywegt [10] point out that decisions based on the incorrect assumption of sufficient minimal repairs could lead to a "spiral down" effect, where system reliability gets worse after repair cycles, i.e., more failures than expected; this happens because the assumed minimal repairs are actually worse than 'good-as-old'. The model we propose in Section 2 allows for a follow-up analysis of whether CM makes system reliability better or worse than it was right before failure.

Kijima [11] proposed a model that includes perfect, minimal, and in-between repairs by introducing the *effective age* of the system after each repair, essentially providing a quantitative measure of whether the repair was successful. A particular case of Kijima's model allowing imperfect repair is considered by Mettas and Zhao [12], who proposed a method to find the maximum likelihood estimates of the model's parameters. Following Kijima [11], Dorado, Hollander, and Sethuraman [13] allow for repairs of varying degree by including so-called known *life supplements*, numbers between zero and one indicating the degree of repair between perfect and minimal. Veber, Nagode and Fajdiga [14] assume one overall life supplement that is unknown, i.e. each repair reduces the effective age of the system by the same fraction $q$. As an extension to a common $q$, Pan and Rigdon [15] allow the repair effectiveness parameter to vary from system to system. Gasmi [16] considers the Weibull distribution in an alternating imperfect repair scheme, i.e. PM followed by CM repeatedly, with common life supplement $q$. Recently Li and Hanson [17] propose to regress the life-supplement of each repair on covariates such as repair type, materials used, et cetera using a Bayesian nonparametric model. Tanwar, Rai, and Bolia [18] review much of the related literature on Kijima-type models.

Our model joins a growing body of literature allowing for differing types of departure from minimal repair, including Kijima [11]. Doyen and Gaudoin [19] consider several classes of imperfect repair models for increasing failure intensities, including models where (a) failure intensity is reduced by a constant factor relative to the current intensity; (b) failure intensity is reduced by a constant factor, but only relative to the most recent repair; and (c) several models based on the virtual age of the system, akin to Kijima's [11] models.

We note that the present article does not provide a method to determine which of CM or PM is optimal for a system at a given point in time. Such decisions critically depend on the context; for instance, maintenance decisions in the context of a nuclear power plant process versus a medical billing records process would be substantially different. Second, the mathematical framework needed to handle such context-specific decisions requires stochastic optimization routines that are outside the scope of the intended aims of this research. However, such routines require information about the underlying probability distribution of the *time until failure* following each of PM or CM. That is, a decision-maker must have sound knowledge of how the system's reliability is affected by maintenance decisions at any given point in time. Dimitrov, Chukova, and Khalil [20] consider the related problem of maintenance costs with imperfect repair, namely warrantee costs within a Kijima Type I model. Garg, Rani, and Sharma [21,22] consider maintenance scheduling for a paper mill assuming a Weibull distribution. Doyen [8] also reviews recent literature on imperfect repair and maintenance scheduling.

## 2. Methodology

The aim of Section 2.1 is to develop a new mathematical framework that encapsulates the impact of management's maintenance decisions on the parameters of the failure-time distribution of repairable systems. In Section 2.2, the failure-time distribution is modeled as a Weibull since, in addition to its wide-spread use in reliability applications, it has desirable theoretical and practical properties that will be highlighted.

### 2.1. A general decision-dependent stochastic process model for repairable systems

Consider a system that is put into operation at time $t_0 = 0$. The time until this system fails has probability density function (pdf) $f(y|\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is a vector of parameters indexing the density from a class such as the Weibull family of distributions. At any time, the system's owner is allowed to perform maintenance of one of two types. In the first type, the system's components are replaced or some other major restoration is performed such that the system's age is reset to zero. This is *preventive maintenance* (PM). The second type of maintenance involves partial repairs or upgrades that do *not* (necessarily) restore the process to an "as good as new" state. This is *corrective maintenance* (CM).

There exists a set of increasing times $\{t_1, \ldots, t_n | t_i < t_{i+1} \forall i = 0, 1, \ldots, n-1\}$ where, at each time, a decision is made to perform either a PM or a CM. The *time series of decisions* is denoted $\{d_i\}_{i=0}^{n}$, where $d_i = 0$ if a PM is performed at time $t_i$ and $d_i = 1$ if a CM is performed; for example, $d_0 = 0$ because we start with a newly restored system. In addition, the time of the most recent PM is denoted $t_i^* = \max\{t_j | j < i, d_j = 0\}$; for example, $t_1^* = 0$ always because $d_0 = 0$. A PM decision at time $t_{i-1}$, i.e., $d_{i-1} = 0$, resets the age of the system to zero. The length of time until the next failure (at time $t_i$) then has pdf $f(t_i - t_i^*|\boldsymbol{\theta}) = f(t_i - t_{i-1}|\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is the parameter vector indexing the density associated with a newly restored system.