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Robust quadratic assignment problem with budgeted uncertain flows

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HIGHLIGHTS

- A robust approach for quadratic assignment problem (RQAP) with budgeted uncertainty.
- An exact and two heuristic methods to solve RQAP.
- Extensive experiments to show performance of methods and quality of solutions.
- RQAP can be solved significantly faster than minmax regret QAP.
- RQAP has adjustable conservativeness while minmax regret QAP has not.

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ABSTRACT

We consider a generalization of the classical quadratic assignment problem, where material flows between facilities are uncertain, and belong to a budgeted uncertainty set. The objective is to find a robust solution under all possible scenarios in the given uncertainty set. We present an exact quadratic formulation as a robust counterpart and develop an equivalent mixed integer programming model for it. To solve the proposed model for large-scale instances, we also develop two different heuristics based on 2-Opt local search and tabu search algorithms. We discuss performance of these methods and the quality of robust solutions through extensive computational experiments.

1. Introduction

[1] introduced the standard quadratic assignment problem (QAP). Standard QAP deals with choosing an optimal way to assign n facilities to n locations to minimize the total material handling cost, given all distances between locations and the amount of material flow between each pair of facilities. A more general form of the QAP was proposed by [2]. [3,4] considered multi-dimensional QAP.

QAP is one of the hardest problems in combinatorial optimization [5,6] and even finding a constant-factor approximate solution for the QAP is NP-hard [7]. However, some specific cases of QAP are easy to solve [8,9]. Many exact and heuristic methods have been developed to solve different cases of QAP. Approximated dynamic programming [10], genetic algorithm [11], parallel algorithms [12,13], hybrid algorithms [14], teaching learning based optimization [15], semidefinite programming relaxations [16,17], mixed integer linear programming reformulation [18–22], reformulation linearization technique (RLT) [23–25], formulation reductions [26], and exploiting data structure [27] are some of these techniques.

QAP has numerous applications such as backboard wiring [28], scheduling problems [29], economic problems [30], designing typewriter keyboards [31], facility layout [32–34], assembling printed circuit boards [35] and many other applications. For a detailed discussion about applications and solution methods for QAP see [36,6,37,38].

In deterministic optimization, it is assumed that input data (e.g. flows between facilities and distances between locations in QAP) are precisely known in advance. Although this assumption can be true in some applications, it is not realistic in many others [39]. [40] proposed a design for robust facility layout under the dynamic demand environment. In their approach, the layout of expected flow or expected demand is applied in all the periods. [41] developed a fuzzy model to address uncertainty in QAP. [42] reviewed facility location problems under uncertainty. [43] studied integration of facility layout design and flow assignment problem under demand uncertainty. [44] considered uncertainty in a hospital layout problem and proposed a robust model for this problem.

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QAP with uncertain locations was studied in [39]. [45] used a robust deviation (minmax regret) approach to deal with uncertainty in material flows.

In uncertain optimization problems with discrete variables, in addition to robust deviation, we can use budgeted uncertainty which has the same complexity as the original model and adjustable conservativeness [46,47]. In practice, for an uncertain mixed integer programming (MIP) problem with interval data, solving robust counterparts for budgeted uncertainty sets is much easier than finding the minmax regret solution. For example, in redundancy allocation problems, this difference is obvious by comparing the results in [48–51], respectively. In addition, former method can find solutions with different levels of conservativeness, while the latter approach outputs only one conservative solution.

In this paper, we consider a generalization of the QAP where the flows are uncertain for some subset J of pairs of facilities. For the flow between each pair of facilities only an interval estimate (uncertainty interval) is available, and the flow can take on any value from the corresponding uncertainty interval. But, for a given protection level $\Gamma \in [0, |J|]$, it is assumed that at most $\lfloor\Gamma\rfloor$ of uncertain flows are allowed to change, and one flow changes by a ratio of at most $(\Gamma - \lfloor\Gamma\rfloor)$ of its uncertainty interval. We are interested in an assignment which minimizes the maximum cost for any possible realization of flows. In other words, because it is unlikely that all uncertain flows adversely affect the cost of assignment, it is assumed that only a subset of uncertain flows change from their nominal values.

This paper is organized as follows. In Section 2, we present notation and problem statement for deterministic and uncertain QAP, and an efficient MIP equivalent for QAP. In Section 3, we develop a mathematical programming formulation of the problem as well as an equivalent MIP model. Then, two heuristic algorithms are described in Section 4. Experimental results are discussed in Section 5, and conclusions are presented in Section 6.

2. Notation and problem statement

In this section, we first present notation and problem statement for classical QAP which is mostly quoted from [45] with some slight adjustments. Then, we present an efficient MIP equivalent for QAP from the literature. Finally, we introduce budgeted uncertainty in flow between facilities and describe some concepts related to the proposed uncertain QAP.

2.1. Classical QAP

In the standard version of QAP, it is assumed that there are n facilities that should be assigned to n locations, in order to minimize the total material handling cost [1]. Let $N = \{1, 2, \ldots, n\}$. For each pair $i, j \in N$ of facilities, let $f_{ij} \geq 0$ be the flow from facility i to facility j. In addition, for each pair $k, l \in N$ of locations, let $d_{kl} \geq 0$ be the travel distance from location k to location l. An assignment of facilities to locations can be represented by an $n \times n$ binary matrix X, where

$$x_{ik} = \begin{cases} 1 & \text{if facility } i \text{ is assigned to location } k, \\ 0 & \text{otherwise.} \end{cases}$$

In any feasible assignment $X \in \{0, 1\}^{n\times n}$, each location must be assigned exactly to one facility, and similarly each facility must be located exactly in one location. Therefore, the set P of all possible assignments is defined by constraints

$$\sum_{k=1}^{n} x_{ik} = 1 \quad \forall i \in N, \tag{1}$$

$$\sum_{i=1}^{n} x_{ik} = 1 \quad \forall k \in N, \tag{2}$$

$$x_{ik} \in \{0, 1\} \quad i, k \in N. \tag{3}$$

For any $X \in P$, let ϕ_i^X denote the location assigned to facility i in assignment X. Let d_{ij}^X be the distance between facilities i and j in assignment X. Therefore,

$$d_{ij}^X := d_{\phi_i^X \phi_j^X} = \sum_{k=1}^{n} \sum_{l=1}^{n} d_{kl} x_{ik} x_{jl}. \tag{4}$$

Given an $n \times n$ flow matrix $f = (f_{ij})$ and an assignment $X = (x_{ik}) \in P$, let $\langle f, X\rangle$ denote the corresponding cost of the assignment,

$$\langle f, X\rangle := \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} f_{ij} d_{kl} x_{ik} x_{jl} = \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} d_{ij}^X. \tag{5}$$

For a given flow matrix f, the *classical* QAP is:

$$\textbf{QAP}(f): \text{ Minimize } \{\langle f, X\rangle \mid X \in P\}. \tag{6}$$

2.2. Xia-Yuan linearization

[2] proposed a more general form of the QAP as follows:

$$\min_{X\in P} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} c_{ijkl} x_{ik} x_{jl} \tag{7}$$

where $c_{ijkl} \geq 0, i, j, k, l \in N$ are given coefficients. Note that **QAP**(f) is a special case of (7) where $c_{ijkl} = f_{ij} d_{kl}$, for all $i, j, k, l \in N$. As discussed in [6], different approaches have been developed to linearize the general QAP (7). [22] demonstrated experimentally that [19] linearization is quite effective as a MIP formulation for the general QAP. The Xia-Yuan linearization for general QAP (7) is:

$$\min_{X\in P} \sum_{i=1}^{n} \sum_{k=1}^{n} (w_{ik} + c_{iikk} x_{ik}), \tag{8}$$

$$w_{ik} \geq \sum_{\substack{j=1\\ j\neq i}}^{n} \sum_{\substack{l=1\\ l\neq k}}^{n} c_{ijkl} x_{jl} - \hat{a}_{ik}(1 - x_{ik}), \quad \forall i, k \in N, \tag{9}$$

$$w_{ik} \geq l_{ik} x_{ik}, \quad \forall i, k \in N, \tag{10}$$

where

$$l_{ik} = \min_{\substack{X\in P\\ x_{ik}=1}} \sum_{\substack{j=1\\ j\neq i}}^{n} \sum_{\substack{l=1\\ l\neq k}}^{n} c_{ijkl} x_{jl}, \quad \text{and}$$
$$\hat{a}_{ik} = \max_{\substack{X\in P\\ x_{ik}=1}} \sum_{\substack{j=1\\ j\neq i}}^{n} \sum_{\substack{l=1\\ l\neq k}}^{n} c_{ijkl} x_{jl}. \tag{11}$$

Observe that constants l_{ik}, $\hat{a}_{ik}$ are obtained by means of solving the regular linear assignment problems (11) which can be done in polynomial time [6]. Values l_{ik} are called Gilmore–Lawler constants [6]. Formulation (8)–(10) has n^2 binary variables, n^2 continuous variables, and $2n^2 + 2n$ linear constraints.

2.3. QAP with budgeted uncertainty

QAP(f) is a valid optimization problem as long as the values of flows and distances are known precisely. However, flows between facilities are typically only estimated within most likely intervals. In the remainder of the paper, we deal with uncertain flows.

Suppose that for any $(i, j) \in N \times N$, two numbers f_{ij}^-, f_{ij}^+ are given, $0 \leq f_{ij}^- \leq f_{ij}^+$, where f_{ij}^- and f_{ij}^+ are lower and upper bounds