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Special Types of Fuzzy Relations

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Abstract

The aim of this paper is to present, in an unitary way, some special types of fuzzy relations: affine fuzzy relations, linear fuzzy relations, convex fuzzy relations, M-convex fuzzy relations. All these fuzzy relations are characterized and we established the inclusions between these classes of fuzzy relations.

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Keywords: fuzzy relations, fuzzy multifunctions, fuzzy multivalued mappings, convex fuzzy processes, affine fuzzy relations, linear fuzzy relations, convex fuzzy relations.

1. Introduction

It is well known that fuzzy relations play an important role in fuzzy modeling and fuzzy control and they also have important applications in relational databases, approximate reasoning, preference modeling, medical diagnosis.

The aim of this paper is to present, in an unitary way, some special types of fuzzy relations: affine fuzzy relations, linear fuzzy relations, convex fuzzy relations, M-convex fuzzy relations, in order to build a fertile ground for application, in further papers, of these fuzzy relations in decision making. All these fuzzy relations are characterized and we established the inclusions between these classes of fuzzy relations.

In this paper X, Y will be vector spaces over $\mathbb{K}$, where $\mathbb{K}$ is $\mathbb{R}$ or $\mathbb{C}$. The concept of fuzzy relation was introduced by L.A. Zadeh in his classical paper[10]. According to L.A. Zadeh a fuzzy relation T between X and Y is a fuzzy set in $X \times Y$, i.e. a mapping $T : X \times Y \to [0, 1]$. We denote by $FR(X, Y)$ the family of all fuzzy relations between X and Y. For $x \in X$ we denote by T_x the fuzzy set in Y defined by $T_x(y) = T(x, y)$. Thus a fuzzy relation can be seen as a mapping $X \ni x \mapsto T_x \in \mathcal{F}(Y)$, where $\mathcal{F}(Y)$ represents the family of all fuzzy sets in Y.

Such mappings were investigated by various mathematicians under different aspects. Thus N. Papageorgiou[6] called this mappings fuzzy multifunctions and studied the continuity of these mappings. E. Tsiporkova, B. De Baets, E. Kerre[9] called this maps fuzzy multivalued mappings and they defined lower and upper semi-continuous fuzzy

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multivalued mapping and the relationships between these two types are studied completely. The continuity of fuzzy multifunctions was also studied by I. Beg[1]. Any application $T : \mathbb{R}^m \to \mathcal{F}(\mathbb{R}^n)$ will be called, by Y. Chalco-Cano, M.A. Rojas-Medar, R. Osuna-Gómez[2] a fuzzy process. A special attention was accorded to convex fuzzy processes. These was introduced by M. Matloka[5] in 2000. Another concept of convex fuzzy process was proposed by Y. Syau, C. Low and T. Wu[8] in 2002. A comparative study of these fuzzy convex processes was made in 2010 by D. Qiu, F. Yang, L. Shu[7]. To avoid any confusion D. Qiu, F. Yang and L. Shu called the former M-convex fuzzy process and the latter SLW-convex process.

2. Preliminary

In this paper, the symbols $\vee$ and $\wedge$ are used for the supremum and infimum of a family of fuzzy sets. We write $\mu_1 \subseteq \mu_2$ if $\mu_1(x) \leq \mu_2(x), (\forall)x \in X$.

Definition 2.1. [3] *Let $\mu_1, \mu_2, \cdots, \mu_n$ be fuzzy sets in X.*
The sum of fuzzy sets $\mu_1, \mu_2, \cdots, \mu_n$ is denoted by $\mu_1 + \mu_2 + \cdots + \mu_n$ and it is defined by

$$(\mu_1 + \mu_2 + \cdots + \mu_n)(x) = \sup_{x_1+x_2+\cdots+x_n=x} \left[\mu_1(x_1) \wedge \mu_2(x_2) \wedge \cdots \wedge \mu_n(x_n)\right] . \tag{1}$$

Let $\mu \in \mathcal{F}(X)$ and $\lambda \in \mathbb{K}$. The fuzzy set $\lambda\mu$ is defined by

$$(\lambda\mu)(x) = \begin{cases} \mu\left(\frac{x}{\lambda}\right) & \text{if } \lambda \neq 0 \\ 0 & \text{if } \lambda = 0, x \neq 0 \\ \vee\{\mu(y) \ : \ y \in X\} & \text{if } \lambda = 0, x = 0 \end{cases} . \tag{2}$$

Definition 2.2. [3] *A fuzzy set $\mu \in \mathcal{F}(X)$ is called linear fuzzy subspace of X if*

1. $\mu + \mu \subseteq \mu$;
2. $\lambda\mu \subseteq \mu, (\forall)\lambda \in \mathbb{K}$.

We de note by $LFS(X)$ the family of all linear fuzzy subspace of X.

Proposition 2.3. [3] *Let $\mu \in \mathcal{F}(X)$. The following statements are equivalent:*

1. $\mu \in LFS(X)$;
2. $(\forall)\alpha, \beta \in \mathbb{K}$, *we have* $\alpha\mu + \beta\mu \subseteq \mu$;
3. $(\forall)x, y \in X$, $(\forall)\alpha, \beta \in \mathbb{K}$, *we have* $\mu(\alpha x + \beta y) \geq \mu(x) \wedge \mu(y)$.

Proposition 2.4. [3] *If $\mu, \rho \in LFS(X)$ and $\lambda \in \mathbb{K}$, then:*

1. $\mu + \rho \in LFS(X)$;
2. $\lambda\mu \in LFS(X)$;
3. $\mu(x) \leq \mu(0), (\forall)x \in X$.

Definition 2.5. [10] *A fuzzy set $\mu \in \mathcal{F}(X)$ is called convex fuzzy set if*

$$\mu(\lambda x_1 + (1 - \lambda)x_2) \geq \mu(x_1) \wedge \mu(x_2),\ (\forall)x_1, x_2 \in X, (\forall)\lambda \in [0, 1] . \tag{3}$$

Definition 2.6. [4] *A fuzzy set $\mu \in \mathcal{F}(X)$ is called fuzzy cone if*

$$\mu(\alpha x) = \mu(x),\ (\forall)x \in X,\ (\forall)\alpha > 0 . \tag{4}$$

A fuzzy set $\mu \in \mathcal{F}(X)$ is called convex fuzzy cone if it is a fuzzy cone which is also a convex fuzzy set.

3. Affine Fuzzy Relations

Definition 3.1. *A fuzzy set $\mu \in \mathcal{F}(X)$ is called affine fuzzy set if*

$$\mu(\lambda x_1 + (1 - \lambda)x_2) \geq \mu(x_1) \wedge \mu(x_2),\ (\forall)x_1, x_2 \in X, (\forall)\lambda \in \mathbb{K} . \tag{5}$$