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# A Mass Conservation Algorithm For Adaptive Unrefinement Meshes Used By Finite Element Methods

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**Abstract**

The Adaptive Hydraulics (ADH) model is an adaptive finite element method to simulate three-dimensional Navier-Stokes flow, unsaturated and saturated groundwater flow, overland flow, and two- or three-dimensional shallow-water flow and transport. In the shallow-water flow and transport, especially involving multispecies transport, the water depth (h), the product of water depth and velocities (uh and vh), as well as water depth and chemical concentration (hc) are dependent variables of fluid-motion simulations and are often solved at various times. It is important for the numerical model to predict accurate water depth, velocity fields, and chemical distribution, as well as conserve mass, especially for water quality applications. Solution accuracy depends highly on mesh resolution. Adaptive mesh refinement (AMR), particularly the h-refinement, is often used to add new nodes in the region where they are needed and to remove others where they are no longer required during the simulation. The AMR is proven to optimize the performance of a computed solution. However, mass with gain or loss can occur when elements are merged due to removing a node at mesh coarsening. Therefore, we develop and implement the mass-conservative unrefinement algorithm to ensure the mass conserved in a merged element in which a node has been removed.

This study describes the use of the Galerkin finite element method to redistribute mass to nodes comprising a merged element. The algorithm was incorporated into the ADH code. This algorithm minimizes mass error during the unrefinement process to conserve mass during the simulation for two-dimensional shallow-water flow and transport. The implementation neither significantly increases the computational time nor memory usage. The simulation was run with various numbers of processors. The results showed good scaling of solution time as the number of processors increases.

*Keywords:* Adaptive mesh, finite element method, mass conservative, shallow-water flow, and ADH model

## 1. Introduction

Accurate solutions are often sought when we solve partial differential equations (PDE) for scientific problems. A variety of numerical errors (e.g., spurious oscillation, numerical spreading, grid orientation, and peak clipping/valley elevating) [1, 2, 3] often exist when we use numerical methods to discretize the PDE system. These errors can be partially mitigated through the implementation of higher order finite element or finite difference discretizations, for example, TVD and ENO methods [4, 5], with the increase of spatial and temporal resolution throughout the


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entire domain to avoid oscillation. But uniformly refining the computational mesh throughout the domain is way too expensive. An adaptive mesh refinement (AMR) approach, which is also called h-refinement, can be more effective in its use of computer resources for large-scale applications. The AMR improves the mesh resolution in regions where the numerical error is large, while keeping the mesh coarse elsewhere. However, the AMR technique is more complex to implement but benefits many applications exhibiting discontinuties, shock waves, or phase changes [6], etc. A wide variety of AMR approaches have been explored in the literature [7, 8, 9, 10, 11, 12], including serial and parallel approaches [13, 14, 15].

Mass conservation is the primary requirement when AMR takes place. In fact, AMR includes two parts: mesh refinement and unrefinement. During the refinement step, it is straightforward to add these nodes and elements in a manner that preserves mass, such as linear interpolation in a linear element. For unrefinement, the nodes are deleted and elements are merged in some regions where a fine mesh resolution has become useless for transient problems. The unrefinement step is necessary to avoid memory and computation overkill. Without special care of transferring data from the unrefined node to its parents, the result often brings up mass conservation issues. Yamoah [16] investigated this problem and proposed three algorithms to restore mass conservation for unrefinement when solving 1-D variably saturated groundwater flow using MATLAB code. These methods are based on either a weighted average approach or the $L_2$ projection approach. He concluded that the mass cannot be preserved, and the mass lost may be significant for a long simulation if a mass-conservative unrefinement algorithm is not used.

We extended the $L_2$ projection algorithm onto 2-D unstructured meshes. We developed the mathematical formulation and implemented the algorithm in the ADaptive Hydraulics (ADH) modeling system—a parallelized numerical model with the following components: saturated and unsaturated groundwater flow, three-dimensional Navier-Stokes flow, two- or three-dimensional shallow-water flow, and the associated transport. When refinement is on the ADH, the element is split if the solution error on the element exceeds the refinement error tolerance. At the added node the data such as water head or chemical concentration are linear interpolation in the element. When the solution error on the element is below the unrefinement error tolerance then the elements are recombined. The node is simply deleted without special care of transferring data from the unrefined node to its parents. In this paper, we first discuss the shallow-water flow application including the governing equations and discretized formulation, numerical methods, and numerical difficulties. We then present the mathematical formulation of the mass-conservative unrefinement algorithm, followed by its implementation. Experimental results are then presented to demonstrate the mass conservation throughout the entire simulation. Finally, we summarize and discuss these results and also present plans for future works.

## 2. ADaptive Hydraulics (ADH)

ADH is a modular, parallel, adaptive finite element model for one-, two-, and three-dimensional flow and transport. ADH simulates saturated-unsaturated groundwater flow, internal flow, and open-channel flow. The groundwater module includes solving 3-D variably saturated groundwater flow and constituent transport. The groundwater flow may be coupled to 2-D diffusive wave or dynamic wave surface water flow and can be loosely coupled to 3-D subsurface heat transport. The internal flow module is used to solve 3-D Navier-Stokes equations. This module is nonhydrostatic and includes a $\kappa - \epsilon$ turbulence model. There are two approaches for open-channel flows: nonhydrostatic and hydrostatic pressure methods. The nonhydrostatic approach is more appropriate for domains near structures where the vertical inertial terms are significant. This approach is the same as that used for the 3-D internal flow, though the free surface is updated dynamically. The hydrostatic approach is to solve the shallow-water flow equations, including 2-D and 3-D variable density flow (baroclinic). The 2-D module includes the capability for wetting-drying, dam-break, supercritical and subcritical flow, sediment transport, and bed change, and also has the capability to include the long-wave vessel effects.

The 2-D shallow-water flow equations are obtained by vertically integrating the mass and momentum under the assumptions of incompressible flow and hydrostatic pressure [17]. Assuming negligible free surface shear and negligible fluid pressure at the free surface, the 2-D shallow-water equations are written as the following partial differential equation (PDE).

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F_x}}{\partial x} + \frac{\partial \mathbf{F_x}}{\partial y} + \mathbf{H} = 0 \tag{1}$$

where $\mathbf{Q} = \{h, uh, vh\}^T$, $t$ is time, $\mathbf{F_x} = \{uh, u^2h + \frac{gh^2}{2} - h\frac{\sigma_{xx}}{\rho}, uvh - h\frac{\sigma_{yx}}{\rho}\}^T$, $\mathbf{F_y} = \{vh, uvh - h\frac{\sigma_{xy}}{\rho}, v^2h + \frac{gh^2}{2} - h\frac{\sigma_{yy}}{\rho}\}^T$,