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Robust Optimization of System Design

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Abstract

The data of real-world optimization problems are usually uncertain, that is especially true for early stages of system design. Data uncertainty can significantly affect the quality of the nominal solution. Robust Optimization (RO) methodology uses chance and robust constraints to generate a robust solution immunized against the effect of data uncertainty. RO methodology can be applied to any generic optimization problem where one can separate uncertain numerical data from the problem's structure. Since 2000, the RO area is witnessing a burst of research activity in both theory and applications.
However, RO could lead to over-conservative requirements, resulting in typical-case bad solutions or even empty solution spaces. This drawback of the classical RO methodology can be overcome by distinguishing between real decision variables and so-called *state* variables. While the first type should satisfy the chance or robust constraints and their value cannot depend on a specific realization of the uncertain data, the state variables are adjustable (i.e., their value can depend on the specific realization of the uncertain data), since most of the constraints defining state variables merely "calculate" their exact value, and hence are always satisfied. In this paper we summarize how adjustable RO approach can be applied to a general uncertain linear optimization problem. Then, using an allocation example we demonstrate how this approach can be integrated in the design optimization process and its impact on the optimal system design.

Keywords: Robust Optimization; State Variables; System Design; Adjustable Robust Counterpart; Uncertainty Set

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1. Introduction

The data of real-world optimization problems more often than not are uncertain – not known exactly at the time the problem is being solved. The reasons for data uncertainty include, among others, measurement and estimation errors coming from the impossibility to precisely measure/estimate the data entries representing the characteristics of physical systems/technological processes/environmental conditions, etc. In addition, implementation errors coming from the impossibility to implement a solution exactly as it is computed, could also be modeled as data uncertainty. In real-world applications of optimization, one cannot ignore cases where a small uncertainty in the data can make the nominal optimal solution completely meaningless. The Robust Optimization (RO) offers a methodology capable of detecting cases when data uncertainty can heavily affect the quality of the nominal solution, and generate a robust solution immunized against the effect of data uncertainty. The goal of RO is to find a robust optimal solution, i.e. find values for decision variables which are feasible for all possible values of uncertain parameters, while optimizing the uncertain objective. By itself, the RO methodology can be applied to any generic optimization problem where uncertain numerical data belonging to a given uncertainty set could be separated from the certain problem structure (i.e, goals, constraints, and decision variables).

The origins of RO date back to the establishment of modern decision theory in the 1950s and the use of the worst case analysis. The paradigm of RO per se, goes back to A.L. Soyster[1] who was the first to consider, as early as in 1973, what now is called Robust Linear Programming. In two subsequent decades there were only two publications on the subject[2,3]. The activity in the area was revived circa 1997, independently and essentially simultaneously, in the frameworks of both Integer Programming[4] and Convex Programming[5,6,7,8]. Since 2000, the RO area is witnessing a burst of research activity in both theory and applications, with numerous researchers involved worldwide.

The standard way to deal with Robust Optimization problem is to find a computationally tractable certain optimization problem called (Approximated) Robust Counterpart (RC), which solution is feasible for the original problem's robust constraints, meets its chance constraints with corresponding probabilities and is (approximately) optimal for its objective. The RO methodology is constraint-wise, i.e. it is applied sequentially per problem constraint. Different modeling techniques and principles are typically applied to robust inequality constraints, robust equality constraints and to chance constraints in order to transform the uncertain problem into its robust counterpart. It should be noted that these special techniques are generally not known to non-experts.

When trying to apply the RO methodology to real life problems we face several challenges, since the classical robust optimization is suitable only where the following set of assumptions hold:

- All decision variables represent "here and now" decisions; they should be assigned specific numerical values as a result of solving the problem before the actual data "reveals itself".
- The decision maker is fully responsible for consequences of the decisions to be made when, and only when, the actual data is within the pre-specified uncertainty set $\boldsymbol{U}$.
- The constraints are *hard*, meaning that we cannot tolerate violations of constraints, even small ones, when the data is in $\boldsymbol{U}$.

However, real life problems often involve state variables, which are not "real" decision variables, thus the first assumption can be relaxed. These variables are called adjustable variables and the constraints containing these variables can be treated differently. The resulting counterpart is called Adjustable Robust Counterpart[9] (ARC). In addition, while the original uncertain design problem can be mixed integer linear problem (MILP), its (approximated) Robust Counterpart could be non-linear. Unfortunately, most of existing optimization solvers are not suitable to solve non-linear problems efficiently. Hence, it is important to find linear formulations or approximations of the uncertain problems counterparts. Another challenge is the formulation of uncertainty sets. In literature, the uncertainty set $\boldsymbol{U}$ is usually considered to be independent of the problem structure and is given in an explicit form. However, in real-life problems the definition of $\boldsymbol{U}$ is not given explicitly and may depend on the value of some of the decision variables. Moreover, real uncertainties can depend on decisions not always explicitly described as decision variables in the original uncertain optimization problem.

Thus, in spite of existing classical techniques, transformation of an uncertain real-life problem to a tractable approximation of its robust counterpart may be a hard and complex process. Consider the following uncertain MILP problem: