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MDEC: MeTiS-based Domain Decomposition for Parallel 2D Mesh Generation

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Abstract

Domain decomposition methods are commonly employed within the context of parallel numerical algorithms. Most often, the domain decomposition is performed before the main computation begins. Within the context of mesh generation, parallel mesh generation is desired when the goal is to mesh a very large geometric domain or if very high accuracy is required. In this paper, we propose a novel technique, which we call the MeTiS-based Domain Decomposition (MDEC) technique, for the decomposition of geometric domains into subdomains for use in parallel 2D mesh generation. Our technique is based upon discrete domain decomposition [1]. The algorithm proceeds by first constructing a background mesh which satisfies a minimum angle constraint of 30 degrees and second partitioning this initial coarse mesh or background mesh into subdomains. Finally, adjustments are applied to the triangles with small boundary angles so that all subdomains in the final decomposition contain boundary angles no smaller than 60 degrees which is a guaranteed property of the domain decomposition algorithm. We prove this guarantee for the boundary angles of the MDEC domain decomposition. Our results show that, in comparison to the medial axis domain decomposition (MADD) algorithm [2], our method provides a better balance of subdomain areas, better boundary angles, and a faster decomposition time. In addition, when the MDEC and MADD subdomains are used in conjunction with a parallel constained Delaunay mesh generation technique (PCDM) [3], the meshes are generated in approximately the same time and have very similar element quality.

Keywords: domain decomposition, mesh partitioning, parallel mesh generation

1. Introduction to Domain Decomposition

The general domain decomposition problem is to decompose the domain of interest into several smaller, non-overlapping domains (called subdomains) based upon some criterion (typically for parallel computation) such as: load balancing, computation requirements, or data dependency. Relevant domains of interest for domain decomposition are: sets, vectors, matrices, or geometries. In this paper, we are concerned with the domain decomposition of geometric domains.

Domain decomposition techniques have been employed in parallel numerical algorithms in order to decompose a large, complex problem into many smaller, simpler subproblems which can be solved in parallel. Within the context

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of parallel numerical algorithms, domain decomposition is typically employed before the main computation begins. For example, domain decomposition methods are often used in the numerical solution of partial differential equations by the finite element method, or other such techniques, in order to decompose the domain into several subdomains on which the PDE is solved. In this example, geometric domains or meshes can be partitioned across the processors so that the numerical PDE can be solved in a distributed manner. Some techniques which have been successfully used for mesh partitioning include: geometric mesh partitioning [4] (which strives to divide the mesh into equal-sized pieces with a small number of interconnecting edges), coorindate bisection [5] (which partitions the vertices of the mesh after projection onto one of the coordinate axes), spectral bisection [6] (which partitions the mesh according to the eigenvectors of the Laplacian of its connectivity graph), and multilevel Kernighan-Lin [7] (which partitions the mesh into a sequence of successively smaller graphs, uses the spectral method to partition the smallest graph, and propagates the partition back through the hierarchy; the Kernighan-Lin method is used to refine the partition). Mesh partitioning remains an active area of research, as various decompositions of the domain can lead to different levels of parallelism in the resulting numerical algorithms.

2. The Domain Decomposition Problem for Parallel Mesh Generation

Despite the fact that domain decomposition is only applied before the main computation step (as described in the previous section), decomposition of the geometry for the purposes of parallel mesh generation is also desired if the size of the geometric domain is very large or if more accuracy is needed in the numerical solution of the PDE. The remainder of this paper focuses on parallel computational techniques for 2D mesh generation.

Parallel mesh generation starts with decomposing the geometric domain into many smaller non-overlapping subdomains. The resulting subdomains are then meshed in parallel. During the mesh generation process, communication between processors may be required in order to preserve the conformality of the overall mesh. However, communication might not be required at all if all the conformal points are predetermined [1].

A review of various parallel mesh generation algorithms is provided in [1]. In that paper, the mesh generation techniques are divided into two categories. The first category of techniques includes mesh generation algorithms for which each subdomain is meshed sequentially. The second category includes techniques for which the degree of coupling between the processors is what defines the degree of communication between the processors in order to preserve conformity of the overall mesh.

A recent domain decomposition algorithm specifically designed and used for parallel mesh generation is the Medial Axis Domain Decomposition (MADD) algorithm [2]. This algorithm decomposes the geometric domain in a divide-and-conquer fashion. The MADD algorithm decomposes the geometric domain by first discretizing the domain boundary. Second, it finds the approximate medial axis of the geometric domain using centroids of the coarse mesh. (This is a boundary conforming Delaunay triangulation of the points created in the previous step). These are actually the nodes of a Voronoi triangulation. Third, it partitions the graph of the Voronoi nodes into two subsets. Fourth, it uses a subset of the Voronoi nodes and connects them to the triangle boundary points to make separators (i.e., segments of the boundary) to separate the two subdomains. Finally, it recursively calls the first four steps using the generated subdomains as inputs until the desired number of subdomains is achieved. For more details on the algorithm, the reader is referred to [2].

In [2], it was noted that using the background mesh directly for the decomposition can lead to small boundary angles. This is undesirable in that the resulting subdomains may lead to less-balanced subdomains and to issues with load balancing within the context of parallel mesh generation. In the next section, we will discuss this further and will describe a way to resolve the small boundary angles. Furthermore, we will use our technique as the basis for a domain decomposition approach for triangular meshes.

3. MeTiS-Based Domain Decomposition (MDEC) with Guaranteed Good Boundary Angles (i.e., Boundary Angles Greater Than 60°)

Our MeTiS-based Domain Decomposition (MDEC) procedure begins with the generation of an initial triangular background mesh on the geometric domain of interest. Next, the mesh is partitioned into the desired number of subdomains. For a given edge of a triangular element, if the edge belongs to elements that belong to different partitions,