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Atomic routing in a deterministic queuing model

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HIGHLIGHTS

- New model for atomic dynamic routing games (bottleneck and makespan/sum objective).
- Polynomial time algorithms for computing maximum and quickest dynamic flows.
- Greedy-type methods compute Nash equilibria in the dynamic game with sum objective.
- Dynamic minimum bottleneck flows and ''narrowest paths'' are NP-hard to compute.
- Nash equilibria for bottleneck objective may not exist, test and decision are NP-hard.

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ABSTRACT

The issue of selfish routing through a network has received a lot of attention in recent years. We study an atomic *dynamic* routing scenario, where players allocate resources with load dependent costs only for some limited time.

Our paper introduces a natural discrete version of the deterministic queuing model introduced by Koch and Skutella (2011). In this model the time a user needs to traverse an edge e is given by a constant travel time and the waiting time in a queue at the end of e. At each discrete time step the first u_e users of the queue proceed to the end vertex of e, where u_e denotes the capacity of the edge e. An important aspect of this model is that it ensures the FIFO property.

We study the complexity of central algorithmic questions for this model such as determining an optimal flow in an empty network, an optimal path in a congested network or a maximum dynamic flow and the question whether a given flow is a Nash equilibrium.

For the bottleneck case, where the cost of each user is the travel time of the slowest edge on her path, the main results here are mostly bad news. Computing social optima and Nash equilibria turns out to be NP-complete and the Price of Anarchy is given by the number of users.

We also consider the makespan objective (arrival time of the last user) and show that optimal solutions and Nash equilibria in these games, where every user selfishly tries to minimize her travel time, can be found efficiently.

1. Introduction

Routing by independent users in a network can be viewed as a non-cooperative game, where selfish players choose their routes through the network (see e.g. [1,2]). From the game-theoretic point of view one can expect the routes chosen by the users to form a *Nash equilibrium* [3] or a strong equilibrium [4], which, in general, does not constitute an overall optimal solution (see [5]). The highest ratio of the social objective of a (strong) Nash equilibrium to the optimal objective is called (strong) Price of Anarchy and gives a measure for the maximum lack of performance that arises due to selfish behaviour (see [6–8]).

The issue of *selfish routing* has received a lot of attention in the recent years, from both the theory as well as the networking communities. Most of the work in the literature has been on static routing, where a user induces load on all edges of her path simultaneously. Recently dynamic models have gained more attention (see [9,10]). Here, each user travels through the network, and at any point in time she only adds load to the edge she is currently using.

We consider a natural discrete version of the deterministic queuing model introduced by Koch and Skutella [11]. Here, each

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edge has a constant travel time and a capacity that states the number of users who can leave an edge per time unit, while the others have to wait until the next point in time. The users leave the edge in the same order as they entered it (FIFO property). In the case that multiple users enter the edge at the same time different tie breaking rules are discussed. The goal is to find a flow with minimum travel time. In a game-theoretic context of a dynamic congestion game, each user chooses a path s.t. her personal travel time is minimized.

Most of our work is on bottleneck problems where the cost of a user is the most expensive resource on her path. There are several applications which motivate this model in the dynamic setting (for more applications of bottleneck games we refer to Banner and Orda [12]).

In the transport of living cargo (livestock) within the European Union there are numerous regulations regarding travel times and resting periods, see [13]. In particular, there are upper limits for the travel times between two stops. At every stop, livestock need to be fed/watered and inspected. In this setting, the links of the network do not correspond to physical roads but rather to aggregated travel routes between two replenishing points, so that a feeding/watering stop is mandatory after the traversal of every arc. The total time spent on a link, i.e., the constant travel time plus the time in the waiting queue, reflects the (aggregated) travel time, where the waiting time in a queue is due to congestion, e.g. at loading docks. Different types of vehicles have different effects on the congestion. Since for instance in extreme weather livestock suffer during transport and waiting, the bottleneck value on a path through the network can be viewed as an indicator of the risk for the animals. Due to the mandatory stops after every link, in fact the maximum travel time of a link is more appropriate here than the total travel time. Even though the links correspond to aggregated routes, still typical rules of traffic networks such as regulations about the right of way apply (e.g. major roads/minor roads, left-before-right-rule). This motivates the study of *local tie-breaking rules* when users enter a link simultaneously.

A second application occurs in routing in all-optical networks. Whenever more than one data packet arrives at a network node at the same time and multiple packets are destined for the same output, blocking occurs. In general, packets are processed in a first-in-first-out (FIFO) matter at every node, but there is also local priority ordering of the input ports associated with the node. Instead of dropping all but one packet, the current state-of-the-art technology uses fibre delay lines (FDLs) to delay the light, where packets circle until they can be forwarded, see e.g. [14–16]. For more information about all-optical networks we refer to standard textbooks such as [17]. There is a natural correspondence to waiting in an FDL and users spending time in the waiting queue in our model. The optical information deteriorates both on the links and in the fibre delay lines, but can be regenerated when the packet is finally forwarded. Since optical regeneration components are expensive (and somewhat slow), the FDLs are typically not equipped with this technology. The overall goal of a "good" routing is to minimize the maximum time between two nodes, the bottleneck value.

A related application occurs in (not necessarily all-optical) communication networks. As mentioned in [12], due to limited size of transmission buffers, there is an interest for the users who are sending data through the network to minimize the utilization of the most utilized buffer in order to avoid deadlocks and reduce packet loss. While Banner and Orda [12] considered the situation that a user permanently uses capacity on her chosen path, we consider a dynamic setting where this capacity is released once the data has left the link.

In a game-theoretic context, each message is sent by a selfish user who attempts to minimize the bottleneck value only on her path. If information from different edges enter a single node and a single edge with low capacity afterwards tie-breaking rules are needed to decide which user's information is processed first. In the networks considered here, the purifier acts on the information from one channel after the other and the order of the messages in one channel remains as it was before.

For other applications of (static) bottleneck games we refer to [12].

1.1. Related work

A deterministic queuing model in the non-atomic case was investigated by Koch and Skutella [11]. Here, each edge has a constant travel time and a capacity which states the amount of flow that can leave an edge per time unit. They show that Nash flows in dynamic congestion games can be computed as special static flows and give results about the Price of Anarchy. Anshelevich and Ukkusuri [18] investigate another non-atomic dynamic routing model, where each edge has a flow dependent latency function. They show that in the single-commodity case there is always a Nash equilibrium which can be computed efficiently.

Atomic models with flow dependent latency functions have been investigated by Farzad et al. [19] and Hoefer et al. [20]. In the first paper different models for the priorities of the users are discussed and the Price of Anarchy for (weighted) atomic models is computed. The second paper deals with the realistic effect that users delay other users after them. Therefore they introduce different local scheduling policies on the edges and show that the existence and computability of equilibria depends on the policy. For example, they show that the existence of equilibria is not guaranteed for every policy. Even for those, where the computation can always be done in polynomial time, a best-response step may be NP-hard to compute.

Work stated above deals with sum objective of the users. We mainly consider the bottleneck objective (see e.g., [12]). In the static case (even weighted) bottleneck games always have an optimal strong equilibrium, since a potential can be constructed from the bottleneck values of the players (see e.g. [21,22]). Furthermore, a Nash equilibrium can be computed in polynomial time in many cases. This can be done with a transformation to a minimum cost flow computation w.r.t. effective costs (see [23]) or by reducing the capacity on the resources with the help of an oracle (e.g., for network games it computes a maximum flow, see [24]). On the other hand, computing an optimal Nash equilibrium is NP-complete in multi-commodity networks (see [12]) or for weighted games (see [25]). The (strong) Price of Anarchy in these games has been investigated by many authors, e.g. by Banner and Orda [12]; Correa et al. [26]; Busch and Magdon-Ismail [27]; and Werth et al. [28]. In [12] it is shown that for linear latency functions the value is bounded from above by the number of edges and that this bound is tight, while for non-linear functions the value is unbounded. In [26] the fairness, i.e., the ratio of the best and the worst objective of a user, is computed, both for congestion and bottleneck games. Busch and Magdon-Ismail [27] showed that the Price of Anarchy is given by the length of a longest path in the network and in [28] it is shown that the strong Price of Anarchy in the unweighted case is given by 2.

1.2. Our contribution

We discuss an atomic, dynamic routing scenario in the deterministic queuing model for sum and bottleneck objectives. For the bottleneck objective we show that deciding whether a flow or path with a desired bottleneck value exists is NP-hard for many classes of instances (even for a single-commodity network with unweighted users). Furthermore, there are instances without a