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Utility Function for modeling Group Multicriteria Decision Making problems as games

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A B S T R A C T

To assist in the decision making process, several multicriteria methods have been proposed. However, the existing methods assume a single decision-maker and do not consider decision under risk, which is better addressed by Game Theory. Hence, the aim of this research is to propose a Utility Function that makes it possible to model Group Multicriteria Decision Making problems as games. The advantage of using Game Theory for solving Group Multicriteria Decision Making problems is to evaluate the conflicts between the decision makers using a strategical approach.

1. Introduction

From time to time individuals face the task of choosing from a set of outcomes that which best meets their preferences on the basis of criteria evaluation. To assist individuals in this process, several methods have been proposed, including those for situations under certainty, such as Linear Programming — LP, Multiobjective Programming — MOP and Multicriteria Decision-Making — MCDM, those under risk, such as Game Theory — GT and Multiattribute Utility Theory — MAUT, and those under the realm of uncertainty, such as Statistics and Simulation.

Within the domain of certainty, the MCDM approach is currently used by individuals in several knowledge areas [1]. However, MCDM may have reduced efficiency due to problems with the aggregation of preferences when the decision-making process involves more than one individual [2,3] and in situations under risk [4]. In this scenario, GT allows to better deal with strategic analysis of group decision-making [5,6].

Some studies have proposed the use of the GT approach for modeling LP and MCDM problems. A pioneer study was performed by Szidarovszky and Duckstein [7], which demonstrates how a multiobjective programming model representing an aquifer management problem can be solved by means of a game theoretical approach. Recently, Madani and Lund [8] proposed modeling MCDM problems as a strategic game, and solved this using non-cooperative GT concept. In their approach, the payoff values are obtained by a transition matrix, which includes both cooperative and non-cooperative outcomes.

However, for generalization of the methodology, a Utility Function – UF – is necessary to translate into a real number all the possible combinations of choices (strategies) in the group MCDM – GMCDM – process. According to Luce and Raiffa [4], the UF would be a reasonable way to describe the preferences of the individual, in order to analyze their choice. Hence, the aim of this research is to propose a UF for modeling GMCDM problems as games.

The UF proposed in this research uses the concept of pairwise comparison in the Euclidian Space to determine the payoffs for all the different strategies of the players. The use of relations in the Euclidian Space has been previously reported by other authors to propose or evaluate MCDM methods [9,10]. Here, the pairwise comparison is an intermediate step for the creation of the UF with the aim of measuring "player satisfaction" [11]. Finally, the UF is applied for modeling the classic game "Battle of the Sexes" and for modeling a travel destination GMCDM problem as a game.

2. The utility function — UF

Let us define a strategic game as $\langle N, A, \succsim_i \rangle$, where N is the set of n players (decision makers), A is the set of m actions (alternatives) and $\succsim_i$ is the preference set over A for each player $i \in N$. In the game proposed here, three strategies for each player are defined, being: (I) keeping the initial alternative when another is offered by another player; (II) changing the initial alternative for the one offered by another player; or (III) changing the initial alternative for another alternative different from that offered by another player. Therefore, the function $\pi\colon \Re_{+}^{c \times n} \rightarrow [0, 1]$, which is a numeric

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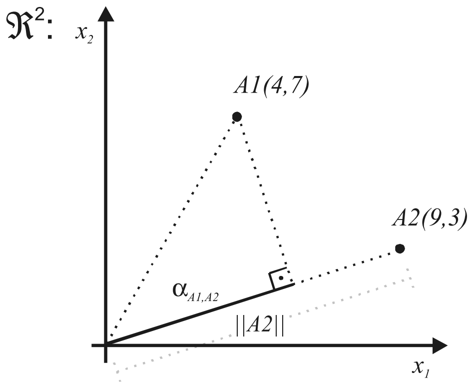

Fig. 1. Scalar projection and the relative measure in the $\Re^2$ Euclidian space.

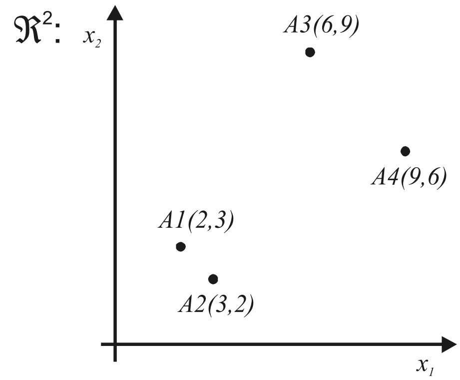

Fig. 2. Example with four alternatives under evaluation against two criteria.

representation of the set of preferences $\succsim_i$ jointly, estimates the payoff for every joint strategy of the n players, considering that a player starts with an alternative $\boldsymbol{x}$ and needs to decide either to keep or to change for another alternative $\boldsymbol{y}$, when it is offered by another player,[1] according to his/her preferences over c criteria.

As an intermediate step for the UF, a Pairwise Comparison Function – PCF φ: $\Re^c_+ \to [0, 1]$, based on the angles and distance (Norm) between the alternatives [9] plotted in the Euclidian space, is proposed. The PCF aims to estimate the subjective pairwise evaluation of decision makers in order to maintain rationality conditions. The PCF proposed here has two main components: (i) a relative component, calculated by the proportional projection of one alternative onto another; and (ii) a direction component, based on the angle between the alternatives. The relative component is calculated using the Scalar Projection ($\alpha_{xy} = \|x\| \cos\theta_{xy}$) of one alternative onto the Norm ($\|y\|$) of another. Fig. 1 illustrates the concept of the relative measure using the Scalar Projection of one alternative $A1$ (defined by the vector [4, 7]) onto the Norm of another alternative $A2$ (defined by the vector [9, 3]), when considering two criteria, x_1 and x_2, in the Euclidian Space $\Re^2$.

In Fig. 1, the comparison of $A1$ (in relation to) and $A2$ is given by the Scalar Projection of $A1$ onto $A2$ ($\alpha_{A1,A2}$) divided by the Norm of $A2$ ($\|A2\|$). In other words, the relative measure is the proportional measurement of how much $A1$ is worth in relation to $A2$, on the $A2$ basis. This is the first component of the PCF, as shown in Eq. (1).

$$\frac{\alpha_{xy}}{\|y\|}. \tag{1}$$

The direction component is proposed based on the angle between the alternatives. This measurement indicates whether the alternatives lie in the same direction, one being when the angle between them is 0° ($\cos 0° = 1$), or in a different direction, varying from zero to one, when the angle between them is more than zero [9]. The purpose of the direction component, which is the angle between the vectors ($\cos\theta_{xy}$), is to incorporate into the value of the PCF a measure of how much the alternative $A1$ is in accordance with the alternative $A2$.

With the components of relation and direction, the PCF, for comparing each pair of alternatives from the set of actions A against the set of criteria C, is defined as shown in Eq. (2).

$$\varphi(x, y) = \left[\frac{\alpha_{xy}}{\|y\|}\right]^{\delta} \cdot \cos\theta_{xy}, \quad \text{where } \delta = \begin{cases} 1, & \text{if } \alpha_{xy} \le \|y\| \\ -1, & \text{otherwise.} \end{cases} \tag{2}$$

Eq. (2) shows the calculations for the PCF proposed, where $\varphi(x, y)$ is the measurement of the pairwise comparison between the alternatives $\boldsymbol{x}$ and $\boldsymbol{y}$ on $\Re^c$ (with c being the number of criteria[2]), $\cos\theta_{xy}$ is the angle between the two alternatives, $\|y\| = \sqrt{y_1^2 + y_2^2 + \cdots + y_c^2}$ is the Norm of the respective vector, and $\alpha_{xy} = \|x\| \cdot \cos\theta_{xy}$ is the Scalar Projection of the vector $\boldsymbol{x}$ onto the vector $\boldsymbol{y}$. The image (range of the function values) varies between zero and one (due to the conditional δ), meaning the closer it is to one the more similar are the alternatives.

In mathematical terms, the PCF satisfies the following properties: (i) $0 \le \varphi(x, y) \le 1$, it establishes values between zero and one for the pairwise comparisons; and (ii) $\varphi(x, y) \ne \varphi(y, x)$, it is asymmetric, i.e., it establishes different values when it has at the beginning one alternative instead of another. A necessary condition is that the criteria must be independent due to the fact that in the Euclidian Space the orthogonality condition is necessary.

In practice, the PCF provides ordinal preference information over the alternatives, which is used to estimate decision makers' pairwise alternative assessment. In fact, the PCF satisfies all properties of preference, that is: (i) comprehensive, since it is possible to compare any pair of alternatives in the Euclidian Space; (ii) it is monotone, since larger values are preferred to smaller values (it is necessary that all criteria be benefiting criteria); (iii) it is reflexive, since if any two alternatives $\boldsymbol{x}$ and $\boldsymbol{y}$ are equal, then $\varphi(x, y) \sim \varphi(y, x)$; and (iv) it is homothetic, since for the same equal two alternatives $\boldsymbol{x}$ and $\boldsymbol{y}$, $k \cdot \varphi(x, y) \sim k \cdot \varphi(y, x)$ for any $k \ge 0$. The transitive property is conditional, given the initial alternative chosen.

To illustrate the preference information provided by the PCF, let us take the example of Fig. 1. In this example, the comparison between the alternative $A1$ and $A2$, given by φ ($A1$, $A2$), is 0.472, whereas the comparison between the alternative $A2$ and $A1$, given by φ ($A2$, $A1$), is 0.654. From these results, it can be induced that $A1 \succ A2$ ($A1$ is preferred to $A2$), because when starting with the alternative $A1$ the PCF value is 0.472 for the comparison with the alternative $A2$, while it is 0.654 when starting with the alternative $A2$ in comparison to the alternative $A1$. Other examples of the PCF preference interpretation can be seen in Table 1, recalling that the closer to one the more similar are the alternatives.

From Examples 1 and 2 of Table 1, one can see that distinguishing the preference information provided by the PCF is necessary. To illustrate this need, let us consider now four alternatives, $A1$, $A2$, $A3$, and $A4$, being evaluated against two criteria, x_1 and x_2, on $\Re^2$. Let us suppose that $A1$ and $A2$ have lower values for these two criteria, while $A3$ and $A4$ have higher values for them, as shown in Fig. 2.

[1] It is considered that a decision always must be made and no player has veto power.

[2] From the calculation of α_{xy} and $\|y\|$ it can be seen that the number of criteria can be straightforwardly changed without structural modifications to the PCF.