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About possible methods of generalization the Beckmann's model, taking into account traffic light modes on the transport graph nodes

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Abstract

The paper describes the original model, which generalizes the Beckmann's model of equilibrium flows, taking into account the delays encountered at the signal-controlled road intersections. Formulated optimization problems of searching of the equilibrium distribution. Formulated and justified the task of optimizing traffic light modes for the transport network in the model.

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1. Description of the model

1.1. Traffic on the transport graph nodes

Most models describing the motion of the vehicle on the road network does not provide dependence of of delays arising from the moving of the network node on flows near it.

Let us remind the concept of «effective number of lanes» which can be illustrated by the following example. Suppose that the vehicle flow, moving along six-lane road arrived to a three-way road intersection (T junction). Assume that the goal of the third of drivers is turning to the left, and the aim of of other is going straight.

In this case, the two leftmost lanes will take vehicles whose goal is to turn left. Thus, at the moment the green light is on for forward movement, the flow of vehicles won't be maximum for the given number of lanes as we might expect, but less by one third, meaning not six thousand but four thousand vehicles per hour.

Let us state the following lemma:

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Lemma on the equilibrium maximum capacity on the signal-controlled road intersection. Consider controlled multilane intersection with Poisson traffic. Consider the flow of cars from one direction at a fixed traffic light phase. Let N — number of incoming lanes, N_i — number of lanes at target road i, S — maximum capacity per lane , $\Omega_i(t)$ — vehicle queue from incoming road to the direction i at time t , $\Omega(t)$ — vehicle queue at incoming road at time t, $\Omega_a(t)$ — vehicle queue from incoming road, which can continue their motion to the direction i on current phase at time t. Then expectation of equilibrium maximum capacity on the signal-controlled road intersection on this phase in time T is:

$$E[S_{max} \cdot T] = \int_0^T min(S \cdot N \cdot \frac{\Omega_a(t)}{\Omega(t)}, \sum_i S \cdot N_i \cdot \frac{\Omega_i(t)}{\Omega(t)})dt.$$

Because of this lemma it was obtained the following consequence:

Corollary of effective number of lanes in the absence of asymmetry queues. Consider controlled multilane intersection with Poisson traffic. Consider the flow of cars from one direction at a fixed traffic light phase. Suppose there is no queue at initial time on the crossroad.

Let q to be intensity of the incoming traffic, k — total intensity of vehicles from the incoming traffic, which can continue their motion on current phase, S — maximum capacity per lane, N — number of incoming lanes, N_i — number of lanes at target road i, k_i — total intensity of vehicles from the incoming traffic, which can continue their motion to the direction i on current phase.

Then expectation of equilibrium maximum capacity on the signal-controlled road intersection on this phase is:

$$E[S_{max}] = \min\left(S \cdot N \cdot \frac{k}{q}, \sum_i S \cdot N_i \cdot \frac{k_i}{q}\right).$$

1.2. Description of the Beckmann's transportation network equilibrium model

Let us describe the Beckmann's transportation network equilibrium model.

Let the city transport network is defined as a directed graph $\Gamma =< V, E >$, where V— set of vertices (crossroads of the city), and $E \subset V \times V$— set of edges (city roads).

Let $W = \{w = (i, j) : i, j \in V\}$ — set of correspondences (in this model, we assume that the sources and sinks coincide with the transport network nodes). Correspondence pair (i, j) means, that vehicle leaves source i and arrives to sink j.

Let P_w — set of paths relevant to correspondence w. $P = \bigcup_{w\in W} P_w$ — the set of all paths. Generally, the number of possible paths increases super-exponentially with increasing number of nodes. This dependence will be close to exponential for real transport systems, but, if we discard all completely unreasonable paths, number of paths increases approximated to quadratic.

x_p — traffic flow on path p. So traffic flow on edge e is $f_e(x) = \sum_{p\in P} \delta_{ep} x_p$, where

$$\delta_{ep} = \begin{cases} 1, & e \in p \\ 0, & e \notin p \end{cases} \tag{1}$$

indicates whether the path p passes through the edge e.

$\tau_e(f_e)$ — delay on edge $(\tau_e{}'(f_e) \geq 0)$;

Let us calculate the total delay on the path p: $G_p = \sum_{e\in E} \tau_e(f_e) \cdot \delta_{ep}$.

Let d_w — the number of correspondences w carried on the transport network at time unit.

$X = \{x \geqslant 0 : \sum_{p\in P} x_p = d_w, w \in W\}$ — set of feasible flows distributions on the paths.

We will minimize delays, though, in general, the delay function can be changed to cost function and depend not only on time but also on fee for toll roads and so on. We can assume that the cost function is increasing and smooth. Also, to bring the problem to problem of convex optimization, later we will use their convexity.