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Multivariate skew normal copula for non-exchangeable dependence

Zheng Wei[a], Seongyong Kim[b], Daeyoung Kim[a,*]

[a]Department of Mathematics and Statistics, University of Massachusetts, Amherst, MA, 01003-9305, USA

[b]Department of Applied Statistics, Hoseo University, Asan-si, Chungcheongnam-do, 31499, Republic of Korea

Abstract

The exchangeability assumption on the dependence structure of the multivariate data is restrictive in practical situations where the variables of interest are not likely to be associated to each other in an identical manner. In this paper, we propose a flexible class of multivariate skew normal copulas to model high-dimensional non-exchangeable dependence patterns. The proposed copulas have two sets of parameters capturing non-exchangeable dependence, one for association between the variables and the other for skewness of the variables. In order to efficiently estimate the two sets of parameters, we introduce the block coordinate ascent algorithm. The proposed class of multivariate skew normal copulas is illustrated using a real data set.

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1. Introduction

The copulas have been increasingly popular for modeling statistical dependence in multivariate data and have been applied to many areas including finance [1], medical research [2], econometrics [3], environmental science [4], actuarial science [5], just to name a few. A key feature of copulas is that they provide flexible representations of the multivariate distribution by allowing for the dependence structure of the variables of interest to be modeled separately from the marginal structure.

Most of the commonly used copulas are exchangeable, which means that the value of the copula is invariant under permutations of its arguments. For some practical situations where one component of the variables influences the other one more than the other way around, exchangeability assumption on copula is not suitable. This is because the dependence based on exchangeable copulas cannot distinguish between components of the variables.

*Corresponding author. Tel.: +1-413-577-0255 ; fax: +1-413-545-1801 .

E-mail address: daeyoung@math.umass.edu

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Recently, several researchers have put considerable efforts into developing multivariate non-exchangeable copulas [6, 7, 8, 9]. The hierarchical Archimedean copulas are constructed from the idea of the compositions of simple Archimedean copulas [8]. However, they requires intricate compatibility conditions and there is no easy stochastic interpretation of them. The vine copulas [7, 10, 11], as popular hierarchical graphical models, utilize the bivariate copulas as the building blocks of a tree structure for multivariate distributions and their dependency structure is determined by a cascade of bivariate copulas. Although the vine copulas are highly flexible, they are not closed under the marginalization and the lack of the stochastic representation makes the interpretation of the vine copulas difficult. The Liouville copulas [9] are proposed as a non-exchangeable generalization of the Archimedean copulas. But, they have some limitations in applications due to unavailability of an algebraically tractable form, strictly positive support, and linear relationship between parameters and dimension.

In this paper we propose a flexible class of multivariate skew normal copulas to model high-dimensional non-exchangeable dependence patterns. The proposed skew normal copula derived from the multivariate skew normal distribution [12, 13] has the two sets of parameters capturing non-exchangeable dependence between the variables of interest, one for correlation between the variables and the other for skewness of the variables. Depending on the restrictions on these parameters, the proposed multivariate skew normal copula produces six parsimonious (non-exchangeable/exchangeable) copulas. We also propose using the block coordinate ascent algorithm to efficiently estimate the parameters in the proposed class of multivariate skew normal copulas. Instead of estimating all parameters simultaneously, the introduced algorithm partitions the parameters into two disjoint blocks, one for the correlation matrix and the other for skewness parameters, and update block by block.

This paper is organized as follows. In Section 2, we briefly review the concept of the multivariate copula and its non-exchangeable property. Section 3 proposes a class of the multivariate skew normal copulas that can capture various (non-exchangeable/exchangeable) dependence structures. In Section 4 we introduce the block coordinate algorithm for estimating the proposed copulas and discuss its convergence property. Section 5 illustrates the proposed class of skew normal copulas with a real data example. We end this article with a discussion in Section 6.

2. Review on multivariate non-exchangeable copulas

We here briefly review the multivariate non-exchangeable copula. For the general copula theory, see [14, 15, 16].

Definition 2.1. *A* $(k+1)$*-dimensional* **copula** *(or* $(k+1)$-**copula***) is a function* $C : [0,1]^{k+1} \mapsto [0,1]$ *satisfying following properties: for* $i = 0,\dots,k$,

(a) $C(u_0, u_1, \dots, u_k) = 0$ *if at least one* $u_i = 0$*;*

(b) $C(1,\dots,1,u_i,1,\dots,1) = u_i$ *for every* $u_i \in [0,1]$*;*

(c) C *is* $(k+1)$*-increasing in the sense that, for any* $J = \prod_{i=0}^{k}[u_i, v_i] \subseteq [0,1]^{k+1}$ *with* $u_i, v_i \in [0,1]$, $volC(J) = \sum_{\boldsymbol{a}} sgn(\boldsymbol{a})C(\boldsymbol{a}) \geq 0$, *where the summation is over all vertices* $\boldsymbol{a}$ *of* J, $\boldsymbol{a} = (a_0, a_1, \dots, a_k)^T$ *is the transpose of* $(a_0, a_1, \dots, a_k)$, *and* $a_i = u_i$ *or* v_i,

$$sgn(\boldsymbol{a}) = \begin{cases} 1, & \text{if} \quad a_i = v_i \quad \text{for an even number of } i's, \\ -1, & \text{if} \quad a_i = v_i \quad \text{for an odd number of } i's. \end{cases}$$

From Definition 2.1, we can see that a $(k+1)$-copula is a joint cumulative distribution function (CDF) on $[0,1]^{k+1}$ with standard uniform marginal distributions.