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An Option Pricing Model using High Frequency Data

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Abstract

We propose a European call option evaluation framework accommodating GARCH-M model and its extension to handle irregularly spaced high-frequency data. The framework takes Bayesian approach to derive the predictive distribution for option prices and their volatilities. These predictive distributions vary as time approaches to the expiry data and provide credibility intervals to evaluate the option market. In empirical study, we illustrate the application using KOSPI200 and its options. Our approach results well-suited in the simulation based option pricing and explains a behavior of option prices close to the expiry.

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Keywords: European Call Option; Volatility; Bayesian; GARCH-M model; MCMC algorithm.

1. Introduction

Today we've entered the era of 'Big Data', so-called the explosion of information, and this trend has deep impact on science, engineering and business. The flood of information enables decision makings to become more and more data-driven, and rigorous data-science to become more increasingly important. In business and finance, various types of Big Data are almost continuously generated daily or at a finer time stamped transaction-by-transaction or tick-by-tick, and corporations utilize such a Big Data to maximize profits by marketing services and to minimize losses by risk management.

High-frequency financial data, observed at a very short-term, have been widely used to study real-time market dynamics, volatility weighting, and strategic behaviour of market participants. However, utilizing high-frequency data also poses substantial modelling challenges, which can be viewed as an example of big data problem. In particular, since practical financial asset trading occurs irregularly in time and hence generates irregularly spaced time series, existing option pricing approaches that assume regularly spaced asset return series are not readily applicable. To mitigate this challenge, we adopt a framework for irregularly spaced time

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series data observed at ultra-high frequency (UHF) referred by [1]. This approach allows us to better utilize information from tick-by-tick asset return series by eliminating the need to aggregate them into regularly-spaced time series.

Volatility is one of the most common risk measure using high-frequency time series. Despite of its imperfections, it is a key factor in option price evaluation and a main interest in risk management. It can be commonly either estimated through the past process of underlying asset price, or implied through a derivative pricing model. Although the implied volatility is popular and common in option pricing, it is derived by unrealistic normal assumption and constant volatility regardless of time. In this respect, historical volatility through generalized autoregressive conditional heteroscedasticity (GARCH) process has been proposed and extended by several researchers [2-8].

However, in real market, practical trading option price generally has shown a gap, sometimes a great difference, from the expectation, and the uncertainty should be considered in the option valuation. For these requirements, Bayesian approach is useful in that predictive distribution enables one not only to forecast option price but also to diagnosis the anomalies of market. A Bayesian GARCH process has been proposed by [9-10], but their suggestions give improper results when distribution of option price is nonsymmetric. Therefore, we propose Bayesian UHF GARCH in mean process, to handle the irregularly spaced high frequency data properly and to reflect the mean process of option pricing model regardless of the distribution assumption.

The remainder of the paper is organized as follows. The Section 2 proposes a Bayesian UHF GARCH-M option pricing model. The Section 3 illustrates empirical results of KOSPI200 call option. The last section concludes the paper with final remarks.

2. A Bayesian UHF GARCH-M Option Pricing Model

Geometric Brownian motion in discrete time assumes a distribution of asset return as $y_{t_i} = \mu\Delta_i + \varepsilon_{t_i}, \varepsilon_{t_i} \sim N(0, \Delta_i\sigma^2)$, for $i = 1, \cdots, T$ and $\Delta_i = t_i - t_{i-1}$. It can be extended as GARCH-M model with σ_{t_i} and $r + \lambda\sigma_{t_i}$ instead of σ and μ, in order to consider risk premium (λ) and conditional heteroscedasticity (σ_{t_i}).

$$y_{t_i} = r\Delta_i + \lambda\Delta_i\sigma_{t_i} + \varepsilon_{t_i}, \varepsilon_{t_i} \sim N(0, \Delta_i) \quad \text{under P-measure,} \tag{1}$$

$$\sigma_{t_i}^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j \frac{\varepsilon_{t_{i-j}}^2}{\Delta_{i-j}} + \sum_{k=1}^{q} \delta_k \sigma_{t_{i-k}}^2 \text{ under P-measure,} \tag{2}$$

The likelihood function of $\mathbf{y} = [y_{t_1}, \cdots, y_{t_T}]$ becomes $L(y_{t_1}, \cdots, y_{t_T}|\lambda, \alpha, \delta) = \prod_{i=1}^{T} \phi\left(\frac{\varepsilon_{t_i}}{\sqrt{\Delta_i}\sigma_{t_i}}\right)$, where $\phi(\cdot)$ is the p.d.f of $N(0,1)$. To satisfy stationary conditions of GARCH model, we assume non-informative normal prior for λ and improper priors for α, δ using $I(\alpha, \delta)$; $\alpha_j > 0, \delta_k > 0$ and $\sum_{j=1}^{p} \alpha_j + \sum_{k=1}^{q} \delta_k < 1$ for $1 \le j \le p, 1 \le k \le q$. Then, the joint prior becomes $p(\lambda, \alpha, \delta) \propto N(\theta_\lambda, \phi_\lambda) \times p(\alpha, \delta) \times I(\alpha, \delta)$, where $N(\theta_\lambda, \phi_\lambda)$ is normal distribution with mean θ_λ and variance ϕ_λ, and the indication function $I(\alpha, \delta)$ is 1 when stationary condition is satisfied, otherwise 0. These priors produce proper posterior distribution of λ, α, δ for given $\mathbf{y}$.

$$f(\lambda, \alpha, \delta|\mathbf{y}) \propto l(\mathbf{y}|\lambda, \alpha, \delta) \times p(\lambda, \alpha, \delta), \tag{3}$$

when $\sigma_{t_i} < \infty$ [9]. It is not possible to sample the marginal distribution directly from the posterior distribution in a closed form, we implement Metropolis algorithm to draw random samples of parameters and make an inference, by proceeding between sampling from its conditional distribution given current values of all other values and updating value of each parameter.