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Optimization of transmission capacities for multinodal markets

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Abstract

This paper considers a homogeneous good competitive market consisting of *n* local markets with given supply and demand functions. The markets are connected by several transmission lines. For every line, the cost functions of transmission capacity increment include fixed and variable components. We set a problem of the total social welfare optimization and discuss its generalization for markets with exporting and importing nodes. We distinguish several cases where the structure of connections corresponds to a tree-type graph and the social welfare function is submodular or supermodular with respect to the set of expanded transmission lines. These properties permit to employ known efficient algorithms that determine the optimal transmission capacities.

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1. Introduction

Markets of natural gas, oil, electricity and other resources play an important role in economies of many countries. An essential component of such markets is a transmission system. Consumers and producers are located at different nodes, and transmission capacities of the lines between the local markets are limited. By recent estimates, the transmission costs may exceed 50% of the electricity price for the industry consumers in Russia. Therefore, the problem of transmission system optimization is of practical interest.

The previous researches on such markets [2, 3, 7, 10, 12, 13] consider primarily models with a fixed network structure. Wilson [14] analyzes market architecture issues for the electric power industry. The recent paper [4] determines the optimal transmission capacity of one line for a two-node market, taking into account transmission losses and costs of transmission line construction. The present study aims to generalize these results for markets with several transmission lines, where the structure of connections corresponds to a tree-type graph. We consider the total welfare optimization problem with an account of the production costs, consumers' utilities and the costs of transmission lines expansion. The optimal solution in this maximization problem determines the total welfare value that can further be reallocated by means of special economic mechanisms letting one determine all Pareto-optimal outcomes. In our model, demand functions reflect possibilities for the welfare increase due to the reduction of the energy prices at local markets. The difficulty of the problem under consideration is that an expansion of

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any line requires valuable fixed costs. If the optimal set of expanded lines was known, the problem would be convex and we could solve it by standard tools. However, the efficient search of this set under a large number of connecting lines requires special methods. In general, the problem is NP-hard [6]. We distinguish several cases where the social welfare function is submodular or supermodular with respect to the set of expanded transmission lines. So the known methods of successive calculations and excluding rules are applicable ([1], [9]).

Below we assume that the transmission system works as if there is a perfect competition among intermediaries who can buy a good at one node and sell it at the other. For many real markets, for instance, electricity markets, the system operators regulate the flows according to this assumption (see [7]). [8] studies the different case and provides a literature review on transmission grid expansion for electricity markets with transmission lines controlled by private transmission companies.

2. Formal model of the market

We consider a homogeneous good market consisting of several local markets and a network transmission system. Let N denote the set of nodes and $L \subseteq N \times N$ be the set of edges.

Every node $i \in N$ corresponds to a local perfectly competitive market. Demand function $D_i(p)$ and supply function $S_i(p)$ characterize respectively consumers and producers in the market and meet the standard conditions: $D_i(p)$ is continuous and monotonically decreasing in p, where $D_i(p) > 0$, and $D_i(p) \to 0$ as $p \to \infty$; $S_i(0) = 0$ and $S_i(p)$ is non-decreasing in p. The demand function relates to the consumption utility function: $U_i(q) = \int_0^q D^{-1}(v)\,dv$. Supply function $S_i(p)$ determines the optimal production volume at node i for the profit maximization problem under a given price: $S_i(p) = Arg\max_v(pv - c_i(v))$, where $c_i(v)$ is the minimal production cost of volume v at node i. Note that the total profit of producers at node i under price $\overline{p}$ is $Pr_i(\overline{p}) = \int_0^{\overline{p}} S_i(p)dp$.

For any $(i, j) \in L$, edge (i, j) represents the transmission line connecting local markets i and j. The line is characterized by the initial transmission capacity Q^0_{ij}, the unit transmission cost e^{ij}_t, the cost function E_{ij} of the transmission capacity increment, including fixed costs e^{ij}_f and variable costs $e^{ij}_v(Q_{ij}, Q^0_{ij})$. For any $(i, j) \in L$, let q_{ij} denote the flow from the market i to market j, $q_{ij} = -q_{ji}$.

Thus, the total transmission costs for edge (i, j) are:

$$E_{ij}\left(q_{ij}\right) = \begin{cases} e^{ij}_f + e^{ij}_v\left(\left|q_{ij}\right|, Q^0_{ij}\right) + e^{ij}_t\left|q_{ij}\right|, & \text{if } \left|q_{ij}\right| > Q^0_{ij}, \\ e^{ij}_t\left|q_{ij}\right|, & \text{if } \left|q_{ij}\right| \leq Q^0_{ij}. \end{cases} \tag{1}$$

In this paper we consider the case where the costs do not depend on the direction of the flow and the flows do not change in time. Then the final transmission capacity Q_{ij} is $|q_{ij}|$, if $\left|q_{ij}\right| > Q^0_{ij}$, otherwise it is Q^0_{ij}. The cost of the line expansion $e^{ij}(Q_{ij}) = e^{ij}_f + e^{ij}_v(Q_{ij}, Q^0_{ij})$ is the overnight construction cost amortized over the life-time T_{ij} of the line using discount rate r: $e^{ij} = r\frac{OC_{ij}}{1-e^{rT_{ij}}}$ (see [11] for the detailed discussion).

We distinguish fixed component e^{ij}_f, assuming that $e^{ij}_v\left(Q^0_{ij}, Q^0_{ij}\right) = 0$ and e^{ij}_v is a monotonous convex function of increment $(Q_{ij} - Q^0_{ij})$. In many practical problems the fixed components are rather substantial, and this determines the complexity of the optimization problem considered below.

Consider the incidence matrix $A = \left\{a_{ij}\right\}$, $i \in N$, $j \in N$ corresponding to the graph G: $a_{ij} = 1$, if $(i, j) \in L$, otherwise $a_{ij} = 0$. Denote $Z(i) = \{j | a_{ij} = 1\}$ the set of nodes connected with node i.

Under any fixed flows of the good $\overrightarrow{q} = \left(q_{ij}, (i, j) \in L\right)$ and production volumes $\overrightarrow{v} = (v_i,\ i \in N)$, the consumption volumes $(\widehat{v_i},\ i \in N)$ are obtained from the balance equations:

$$\widehat{v_i} = v_i + \sum_{j \in Z(i)} q_{ji},\ i \in N.$$

The total social welfare for the network market is the total consumption utility over the market minus the total production and transmission costs:

$$\overline{W}(\overrightarrow{q}, \overrightarrow{v}) = \sum_{i \in N} [U_i\left(v_i + \sum_{l \in Z(i)} q_{li}\right) - c_i(v_i)] - \sum_{(i,j) \in L,\ i<j} E_{ij}\left(q_{ij}\right). \tag{2}$$