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Fuzzy TOPSIS: A General View

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Abstract

The aim of this survey paper is to offer a general view of the developments of fuzzy TOPSIS methods. We begin with a literature review an we explore different fuzzy models that have been applied to the decision making field. Finally, we present some applications of fuzzy TOPSIS.

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1. Introduction and literature review

The problems of Multi-Criteria Decision Making (MCDM) appear and are intensely applied in many domains, such as Economics, Social Sciences, Medical Sciences etc. Sometimes, MCDM problems are mentioned as Multiple-Criteria Decision Analysis (MCDA) or Multi-Attribute Decision-Making (MADM) (see [22, 27, 47, 60]). In spite of their diversity, the MCDM have as common characteristic multiple objectives and multiple criteria which usually are in conflict with each other. The decision makers have to select, assess or rank these alternatives according to the weights of the criteria. In the last decades the MCDM techniques have become an important branch of operations research (see [23, 46, 65]).

In many real-world situations, the problems of decision making are subjected to some constraints, objectives and consequences that are not accurately known. After Bellman and Zadeh [8] introduced for the first time fuzzy sets within MCDM, many researchers have been preoccupied by decision making in fuzzy environments. The fusion between MCDM and fuzzy set theory has led to a new decision theory, known today as fuzzy multi-criteria decision making (FMCDM), where we have decision-maker models that can deal with incomplete and uncertain knowledge and information. The most important thing is that, when we want to assess, judge or decide we usually use a natural language in which the words do not have a clear, definite meaning. As a result, we need fuzzy numbers to express linguistic variables, to describe the subjective judgement of a decision maker in a quantitative manner. Fuzzy numbers (FN) most often used are triangular FN, trapezoidal FN and Gaussian FN.

We highlight that the concept of linguistic variable introduced by Zadeh in 1975 (see [61]) allows computation with words instead of numbers and thus linguistic terms defined by fuzzy sets are intensely used in problems of decision theory for modeling uncertain information. There are very good monographs (see for instance [17]) and surveys papers [1, 12, 25, 33, 38] on FMCDM. Recently, some new methods have been explored [3, 53, 67].

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After Atanassov [4] introduced the concept of intuitionistic fuzzy sets, where each element is characterized by a membership function, as in fuzzy sets, as well as by a non-membership function, the interest in the study of the problems of decision making theory with the help of intuitionistic fuzzy sets (see [11, 26, 29, 31, 32, 62]) has increased.

As a generalization of the concept of the classic set, fuzzy set, intuitionistic fuzzy set etc., Smarandache [42] firstly proposed the concept of neutrosophic set. In paper [49] there are proposed set-theoretic operators on an instance of neutrosophic set called interval neutrosophic set. Recently, neutrosophic sets have been applied in MCDM (see [36, 56, 57, 58, 63]).

Torra and Narakawa [45] and Torra [44] introduced the concept of hesitant fuzzy set, which undergoes a much more flexible approach for decision makers when they provide their decisions. Therefore, hesitant fuzzy sets have become useful in MCDM problems [37, 39, 48, 52].

The aim of this survey paper is to offer a general view of the developments of fuzzy TOPSIS methods. We begin with a literature review an we explore different fuzzy models that have been applied to the decision making field. Finally, we present some applications of fuzzy TOPSIS.

2. Basic concepts and definitions

Definition 2.1. *[21] A fuzzy number (FN) is a fuzzy set in $\mathbb{R}$, namely a mapping $x : \mathbb{R} \to [0,1]$, with the following properties:*

1. *x is convex, i.e. $x(t) \geq \min\{x(s), x(r)\}$, for $s \leq t \leq r$;*
2. *x is normal, i.e. $(\exists)t_0 \in \mathbb{R} \ : \ x(t_0) = 1$;*
3. *x is upper semicontinuous, i.e.*

$$(\forall)t \in \mathbb{R}, (\forall)\alpha \in (0,1] \ : \ x(t) < \alpha, (\exists)\delta > 0 \text{ such that } |s-t| < \delta \Rightarrow x(s) < \alpha \,. \tag{1}$$

Remark 2.2. *Among the various types of FNs, triangular FNs and trapezoidal FNs are the most popular. A triangular FN is defined by its membership function*

$$x(t) = \begin{cases} 0 & \text{if} \quad t < a \\ \frac{t-a}{b-a} & \text{if} \quad a \leq t < b \\ \frac{c-t}{c-b} & \text{if} \quad b \leq t < c \\ 0 & \text{if} \quad t > c \end{cases} \text{, where } a \leq b \leq c \,, \tag{2}$$

and it is denoted $\tilde{x} = (a,b,c)$. A trapezoidal FN is defined by its membership function

$$x(t) = \begin{cases} 0 & \text{if} \quad t < a \\ \frac{t-a}{b-a} & \text{if} \quad a \leq t \leq b \\ 1 & \text{if} \quad b < t < c \\ \frac{d-t}{d-c} & \text{if} \quad c \leq t \leq d \\ 0 & \text{if} \quad t > d \end{cases} \text{, where } a \leq b \leq c \leq d \,, \tag{3}$$

and it can be expressed as $\tilde{x} = (a,b,c,d)$.

Remark 2.3. *[14, 22, 24] Let $\tilde{x} = (a_1,b_1,c_1), \tilde{y} = (a_2,b_2,c_2)$ be two non negative triangular FNs and $\alpha \in \mathbb{R}_+$. According to the extension principle, the arithmetic operations are defined as follows:*

1. *$\tilde{x} + \tilde{y} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$;*
2. *$\tilde{x} - \tilde{y} = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$;*
3. *$\alpha\tilde{x} = (\alpha a_1, \alpha b_1, \alpha c_1)$;*
4. *$\tilde{x}^{-1} \cong (1/c_1, 1/b_1, 1/a_1)$;*
5. *$\tilde{x} \times \tilde{y} \cong (a_1 a_2, b_1 b_2, c_1 c_2)$;*
6. *$\tilde{x}/\tilde{y} \cong (a_1/c_2, b_1/b_2, c_1/a_2)$.*

We note that the results of (4) – (6) *are not triangular FNs, but they can be approximated by triangular FNs.*

Remark 2.4. *[17, 28, 59] Let $\tilde{x} = (a_1,b_1,c_1,d_1), \tilde{y} = (a_2,b_2,c_2,d_2)$ be two non negative trapezoidal FNs and $\alpha \in \mathbb{R}_+$. The arithmetic operations are defined as follows:*

1. *$\tilde{x} + \tilde{y} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$;*
2. *$\tilde{x} - \tilde{y} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$;*
3. *$\alpha\tilde{x} = (\alpha a_1, \alpha b_1, \alpha c_1, \alpha d_1)$;*
4. *$\tilde{x}^{-1} \cong (1/d_1, 1/c_1, 1/b_1, 1/a_1)$;*