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Cluster analysis of Diffusion Tensor fields with application to the segmentation of the Corpus Callosum

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Abstract

Accurate segmentation of the Corpus Callosum (CC) is an important aspect of clinical medicine and is used in the diagnosis of various brain disorders. In this paper, we propose an automated method for two and three dimensional segmentation of the CC using diffusion tensor imaging. It has been demonstrated that Hartigan's K-means is more efficient than the traditional Lloyd algorithm for clustering. We adapt Hartigan's K-means to be applicable for use with the metrics that have a f-mean (e.g. Cholesky, root Euclidean and log Euclidean). Then we applied the adapted Hartigan's K-means, using Euclidean, Cholesky, root Euclidean and log Euclidean metrics along with Procrustes and Riemannian metrics (which need numerical solutions for mean computation), to diffusion tensor images of the brain to provide a segmentation of the CC. The log Euclidean and Riemannian metrics provide more accurate segmentations of the CC than the other metrics as they present the least variation of the shape and size of the tensors in the CC for 2D segmentation. They also yield a full shape of the splenium for the 3D segmentation.

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1. Introduction

The corpus callosum (CC) is a great fiber bundle in the white matter of the brain. It connects the two hemispheres of the brain. An accurate segmentation of the CC is needed in clinical applications (e.g. surgical planning and disease prognosis). There is a large quantity of research that has used Magnetic Resonance Imaging (MRI) to segment the CC. Diffusion Tensor Imaging (DTI) is an advanced technique of MRI which is able to measure the diffusion of the water inside the brain tissues. DTI has also been used for the segmentation of the CC (e.g.[1–4]). Lenglet et al.[1] proved that the segmentation of the CC using the Riemannian metric is superior to the segmentation using Euclidean metric. Their method was based on *surface evolution* which seeks the optimal partition through a Bayesian formulation. Lee et al.[2] proposed an automatic 2D segmentation of the CC using the color coded map of the diffusion tensors. Goh and Vidal[3] proposed the Locally Linear algorithm for diffusion tensor clustering (LLDTC) to segment the fiber bundles (e.g. Cingulum and the CC) using Riemannian and log-Euclidean metrics. In fact, LLDTC is a

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generalization of a locally linear embedding method which is a dimensionality reduction method based on embedding high dimensional data on Euclidean space into low dimensional space. The LLDTC uses the K-means algorithm to cluster the embedded points. Nazem-Zadeh et al.[4] proposed a three dimensional segmentation of the CC using DTI. They used the diffusivity pattern of the CC as prior information. A similarity measure, based on a *speed function*, has been proposed to segment the CC and its subdivisions.

Non-Euclidean means are alternatives to the Euclidean mean for the space of covariance matrices. In the context of tensor interpolation, the linear interpolation of tensors (using the Euclidean mean) produces tensors with parabolic, non monotonic, determinants and hence large sized tensors in the interpolated path[5]. As opposed to the Euclidean mean, the log Euclidean[5] and Riemannian[6] means yield monotonic determinants of the interpolated tensors. Carmichael et al.[7] stated that using the log Euclidean and Riemannian means are more effective in smoothing the tensor field in anisotropic regions than the Euclidean mean. Cholesky, Procrustes size and shape and root Euclidean means are other non-Euclidean alternatives for interpolating and smoothing diffusion tensors[8]. For more details about non-Euclidean methods see[8,9].

In K-means cluster analysis, Hartigan's method[10] (also known as the Exchange method[11]) is more efficient than the Lloyd algorithm[12], as it results with tighter clusters than the Voronoi diagram[13] and hence it converges to a solution with smaller within cluster sum of squares (WCSS). The Lloyd method considers how close an object is to the centroid of the cluster and hence each object is assigned to its closest centroid. The result of Hartigan's method guarantees that no movement of an object to any cluster will reduce the WCSS.

In this article, in Section 2, we present the objective function for the Lloyd algorithm and Hartigan's method. In Section 3, we show that Hartigan's method can be generalized to the metrics which have the f-mean property. In Section 4 we provide a 2D and 3D segmentation of the CC. We show that the log Euclidean and Riemannian metrics outperform the other metrics in the segmentation of the CC as they provide the least size and shape variations of the tensors in the CC for 2D segmentation, and they segment the splenium (posterior end) as part of the CC for 3D segmentation.

2. Hartigan's method

We begin by introducing some basic notations that are used in this section. Suppose $\mathbf{A} = \{\mathbf{A}_1, \dots, \mathbf{A}_M\}$ is a set of covariance matrices. Suppose $C = \{C_1, \dots, C_K\}$ where C_k, for $k \in \{1, \dots, K\}$, is the set of the indices of the covariance matrices inside the cluster k (i.e. C is a set of sets of the indices arising from a partition of A into K clusters). Let $\bar{\mathbf{A}}_p$ be the centroid of the cluster p and $m(p)$ be the number of covariance matrices in the cluster p.

Both Hartigan's method and Lloyd's algorithm aim to find a partition of a data set that minimizes the WCSS, but they use different methods to reduce the WCSS. The objective of Lloyd's algorithm is to find a partition C that minimizes the WCSS, i.e. to $\arg\min_{C} \sum_{k=1}^{K} \sum_{i \in C_k} d^2(\mathbf{A}_i, \bar{\mathbf{A}}_k)$, where $d(\mathbf{A}_i, \bar{\mathbf{A}}_k)$ is the distance between $\mathbf{A}_i$ and $\bar{\mathbf{A}}_k$, where $\bar{\mathbf{A}}_k$ is the mean of the covariance matrices in the cluster k. Hartigan's method iterates over all the covariance matrices in $\mathbf{A}$ to draw each covariance matrix out of its cluster, updates the centroid of the clusters, and assigns the covariance matrix to another cluster if doing so will reduce the WCSS. Suppose C_p and $C_j \in C$ and let $i \in C_p$. The method tries to move $\mathbf{A}_i$ into cluster j where $j \in \{1, \dots, k\}$ and $j \neq p$, then calculates the new WCSS. Suppose the partitioning before the movement was $C = \left\{C_1, \dots, C_p, \dots, C_j, \dots, C_K\right\}$. Then the partitioning after the movement is $C = \left\{C_1, \dots, C_p \setminus \{i\}, \dots, C_j \cup \{i\}, \dots, C_K\right\}$. After the movement, we need to calculate the new centroids and the new WCSS. Spath[11] showed that, using the Euclidean distance, the new centroids and the new WCSS can be calculated in a simple and quick way as a function of the previous centroids and the previous WCSS respectively. Let $\bar{\mathbf{A}}_{np}$ be the new centroid of the cluster p after the movement of $\mathbf{A}_i$ from the cluster p. Let $C_{np} = C_p \setminus \{i\}$ and $C_{nj} = C_j \cup \{i\}$. Then WCSS of the clusters p before the movement is $WCSS_p = \sum_{t \in C_p} \left\|\mathbf{A}_t - \bar{\mathbf{A}}_p\right\|^2$. The new WCSS of the cluster p and j when $\mathbf{A}_i$ is moved from the cluster p to j are $WCSS_{np} = WCSS_p - \frac{m(p)}{m(p)-1} \left\|\mathbf{A}_i - \bar{\mathbf{A}}_p\right\|^2$ and $WCSS_{nj} = WCSS_j + \frac{m(j)}{m(j)+1} \left\|\mathbf{A}_i - \bar{\mathbf{A}}_j\right\|^2$ (see[11]). As the movement is between the cluster p and the cluster j, the WCSS of other clusters will not be affected. The new total of the WCSS over all clusters is $WCSS_{new} = WCSS_{pre} + \frac{m(j)}{m(j)+1} \left\|\mathbf{A}_i - \bar{\mathbf{A}}_j\right\|^2 - \frac{m(p)}{m(p)-1} \left\|\mathbf{A}_i - \bar{\mathbf{A}}_p\right\|^2$ where $WCSS_{pre}$ is the total of WCSS, over all clusters, before the movement. Let $G_j = \frac{m(j)}{m(j)+1} \left\|\mathbf{A}_i - \bar{\mathbf{A}}_j\right\|^2 - \frac{m(p)}{m(p)-1} \left\|\mathbf{A}_i - \bar{\mathbf{A}}_p\right\|^2$, then for the movement to be successful $\exists j \in \{1, \dots, k\}$ such that $G_j < 0$. In this case the movement of $\mathbf{A}_i$ will be to the cluster j with smallest G_j as we want the largest reduction in the $WCSS_{new}$. Therefore, the objective function for Hartigan's method is $\min_{j \neq p, G_j < 0} G_j$.