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2-Power Domination in Certain Interconnection Networks

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Abstract

The k-power domination problem generalizes domination and power domination problems. The k-power domination problem is to determine a minimum size vertex set $S \subseteq V(G)$ such that after setting $X = N[S]$ and iteratively adding to X vertices x that have a neighbour v in X such that at most k neighbours of v are not yet in X till we get $X = V(G)$. The least cardinality of such set is called the k-power domination number of G and is denoted by $\gamma_{p,k}(G)$. In this paper, we restrict our discussion to $k = 2$, referred to as 2-power domination. We compute 2-power domination number for certain interconnection networks such as hypertree, sibling tree, X-tree, Christmas tree, mesh, honeycomb mesh, hexagonal mesh, cylinder, generalized Petersen graph and subdivision of graphs.

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Keywords: 2-power domination; hypertree; hexagonal mesh; honeycomb mesh; cylinder; Christmas tree.

1. Introduction

Electric power companies need to continually monitor the state of their systems as in the case of voltage magnitude at loads and machine phase angle at generators. One method of monitoring these variables is to place Phase Measurement Units, called PMUs, at selected locations in the system. The problem is to locate a smallest set of PMUs to monitor the entire system. In electric power system, a vertex represents an electric node and an edge represents a transmission line joining two electrical nodes[1].

A set $S \subseteq V$ is a dominating set in a graph $G(V, E)$ if every vertex in $V \backslash S$ has at least one neighbour in S, that is $N[S] = V$. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of dominating sets of G. The power domination problem is considered as a variation of the dominating set problem. We define a set S to be a power dominating set (PDS) if every vertex in G is observed by S. The k-power domination is a generalization of domination and power domination problems. The k-power domination of G, denoted by $\gamma_{p,k}(G)$, is the minimum cardinality of a k-power dominating set of G. For any graph G, $1 \leq \gamma_{p,k}(G) \leq \gamma_p(G) \leq \gamma(G)$.

Generalized power domination number of any connected graph G of order n, satisfies $\gamma_{p,k}(G) \leq \frac{n}{k+2}$. Also for any claw-free $(k + 2)$-regular graph of order n, $\gamma_{p,k}(G) \leq \frac{n}{k+3}$[2]. Generalized power domination has been well studied for regular graphs[3], Sierpinski graphs[4], trees[1], interval graphs, circular-arc graphs[5], grid[6], claw-free[7], block

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graphs[8], product graphs[9], cylinder, torus and generalized Petersen graph[10]. Moreover, the power domination number $\gamma_p(G) \leq n/3$ for any graph G of order $n \geq 3$.

Definition 1.1. [1] *For $v \in V(G)$, the open neighbourhood of v, denoted as $N_G(v)$, is the set of vertices adjacent to v; and the closed neighbourhood of v, denoted by $N_G[v]$, is $N_G(v) \cup \{v\}$. For a set $S \subseteq V(G)$, the open neighbourhood of S is defined as $N_G(S) = \bigcup_{v \in S} N_G(v) \backslash S$ and the closed neighbourhood of S is defined as $N_G[S] = N_G(S) \cup S$. For brevity, we denote $N_G[S]$ by $N[S]$.*

Definition 1.2. [1] *For a graph $G(V, E)$, $S \subseteq V$ is a dominating set of G if every vertex in $V \backslash S$ has at least one neighbour in S. The domination number of G, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set of G.*

Definition 1.3. [2] *Let $G(V, E)$ be a graph and let $S \subseteq V(G)$. For $k \geq 0$, we define the sets $M^i(S)$ of vertices monitored by S at level i, $i \geq 0$, inductively as follows:*

1. $M^0(S) = N[S]$.
2. $M^{i+1}(S) = \cup\{N[v] : v \in M^i(S)$ *such that* $|N[v] \backslash M^i(S)| \leq k\}$.

Note that $M^i(S) \subseteq M^{i+1}(S) \subseteq V(G)$ for any i. Moreover, every time a vertex of the set $M^i(S)$ has at most k neighbours outside the set, we add its neighbours to the next generation $M^{i+1}(S)$. If $M^{i_0}(S) = M^{i_0+1}(S)$ for some i_0, then $M^j(S) = M^{i_0}(S)$ for any $j \geq i_0$. We thus define $M^\infty(S) = M^{i_0}(S)$.

Definition 1.4. [2] *Let $G = (V, E)$ be a graph, let $S \subseteq V(G)$, and let $k \geq 0$ be an integer. If $M^\infty(S) = V(G)$, then the set S is called a k-power dominating set of G, abbreviated kPD-set. The minimum cardinality of a kPD-set in G is called the k-power domination number of G written $\gamma_{p,k}(G)$.*

In this paper, we restrict our discussion to $k = 2$. In general, the k-power domination problem is NP-complete[2]. In fact, the problem has been shown to be NP-complete even when restricted to bipartite graphs and chordal graphs[1].

2. Main Results

In this section, we solve 2-power domination problem for certain interconnection networks.

A tree is a connected graph that contains no cycles. The most common type of tree is the binary tree. It is so named because each node can have at most two descendants. A binary tree is said to be a complete binary tree if each internal node has exactly two descendants. These descendants are described as left and right children of the parent node. Binary tree are widely used in data structures because they are easily stored, easily manipulated, and easily retrieved. Also many operations such as searching and storing can be easily performed on tree data structures. Furthermore, binary trees appear in communication pattern of divide-and-conquer type algorithms, functional and logic programming, and graph algorithms. A rooted tree represents a data structure with a hierarchical relationship among its various elements.

Definition 2.1. *Let T be the tree formed from a star $K_{1,n}$ and identifying each of its pendant vertices with binary trees; that is T has at most one vertex of degree 4 or more. We call such a tree T an extended spider tree and denote it by T_n^*. The vertex of degree n in $K_{1,n}$ is said to be at level 0 in the extended spider. Its neighbours which are n is number are at level 1. Their descendents are in level 2 and so on.*

Theorem 2.2. *Let G be a tree, then $\gamma_{p,2}(G) = 1$ if and only if G is an extended spider or a binary tree.*

Proof. *Let us assume that G is an extended spider tree T^*. Consider $S = \{v : deg(v) > 3\}$. Then v is the root vertex and $|S| = 1$. Now $M^0(S) = N[S] = \{v, x_i/x_i \text{ is the root of a binary tree}, 1 \leq i \leq n\}$. In other words, $M^0(S)$ contains all the vertices at level 0 and level 1 of the extended spider. Since every vertex at level 1 has exactly two children, $M^1(S)$ includes all the vertices at level 2 and no more. Thus at every step, $M^i(S)$ includes all the vertices in level $i + 1$, $1 \leq i \leq r - 1$ where r is the maximum height of the binary trees identified with vertices of $K_{1,n}$. Thus $M^{r-1}(S) = V(G)$. Therefore, $\gamma_{p,k}(G) = 1$. Further if G is a binary tree, then by Theorem 2.5, $\gamma_{p,k}(G) = 1$.*

Conversely, let us assume that $\gamma_{p,2}(G) = 1$. Let $S = \{v\}$ be a 2-power dominating set of G. Consider $M^0(S) = N[v]$.