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Hypergraph grammar based adaptive linear computational cost projection solvers for two and three dimensional modeling of brain

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Abstract
In this paper we present a hypergraph grammar model for transformation of two and three dimensional grids. The hypergraph grammar concerns the proces of generation of uniform grids with two or three dimensional rectangular or hexahedral elements, followed by the proces of h refinements, namely breaking selected elements into four or eight son elements, in two or three dimensions, respectively. The hypergraph grammar presented in this paper expresses also the two solver algorithms. The first one is the projection based interpolation solver algorithm used for computing H^1 or L^2 projections of MRI scan of human head, in two and three dimensions. The second one is the multi-frontal direct solver utilized in the loop of the Euler scheme for solving the non-stationary problem modeling the three dimensional heat transport in the human head generated by the cellphone usage.

Keywords: hypergraph grammar, projection solver, MRI scan, non-stationary heat transfer, brain heating by cellphone

1 Introduction

The two and three dimensional h adaptive finite element method (FEM) [12, 4] is the sophisticated tool for performing numerical simulations [2, 1, 19, 7]. In this paper we present a hypergraph grammar model expressing the h adaptive mesh transformations of two and three dimensional grids with rectangular and hexahedral elements.

In our previous work we modeled the two dimensional triangular and rectangular grids [16, 15, 14, 13] as well as three dimensional hexahedral grids [17, 18] by CP-graph grammars. Hypergraphs and hypergraph grammar were originally introduced by [9, 10] as Hyperedge Replacement Grammar . The hypergraph grammars were introduced as their extension by [20] for modeling transformations of two dimensional adaptive grids with rectangular elements.

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The hypergraph grammars have been also recently used for modeling of the multi-frontal direct solver algorithm executed over two dimensional rectangular elements grids [8]. In this paper we present an extension of [20] for modeling of transformations of three dimensional adaptive grids composed with hexahedral elements.

The projection-based interpolation (PBI) [3] is a linear computational cost algorithm for solving the projection problem over the refined grids. It has been already tested on two and three dimensional examples [19, 7] In this paper we model the projection solver implementing the projection based interpolation algorithm. The projection solver is used for computing the H^1 projection of the MRI scan data of the human head.

The second solver modeled by hypergraph grammar is the multi-frontal direct solver algorithm [5, 6] solving the non-stationary heat transfer problem over the human head, the heating induced by the cellphone usage. The solver is executed in a loop, for each time step, utilized in the Euler scheme. It should be emphesized that the multi-frontal solver for such adaptive grid usually has computational cost varying between $\mathcal{O}(N)$ and $\mathcal{O}(N^2)$ depending on the topology of the mesh. We also discuss the advantages and disadvantages of using the hypergraphs instead of CP-graphs.

We conclude the paper with the application of the hypergraph grammar based projection solver for modeling of heating of the human head enforced by electromagnetic waves generated by the cellphone.

2 Hypergraph grammar for modeling two dimensional adaptive mesh transformations

In this section we present the hypergraph grammar productions for generation and adaptation of two dimensional meshes with rectangular elements. The productions are summarized in Figure 1, and they have been obtained by modification of the productions presented in [20]. We start with the graph grammar productions that can be used for both sequential and parallel generation of the initial mesh with point singularity located at the center of the bottom of the mesh. We start with executing production **(P init)** that transforms the initial state **(S)** into the initial mesh. Next, we proceed with refinements of the left and right element, by executing the productions **(P init left)** and **(P init right)**, followed by the execution of the production **(P irregularity)**. In order to generate further local refinements of the mesh, we proceed with productions **(P break interior)** and **(P enforce regularity)**, executed one after another, as many times as we need to have levels of refinement.

3 Hypergraph grammar for modeling three dimensional adaptive mesh transformations

The process of generation of the three dimensional computational mesh with hexahedral elements starts with execution of the **(P init** production, presenting in Figure 2, generating the hypergraph representing a single finite element. In case of uniform mesh adaptations, we can prepare a sequence of graph grammar productions replacing the single element by a uniform cluster of elements. The exemplary production **(P init break)** presented in Figure 3 generates the uniform mesh of $2 \times 2 \times 2$ elements. In order to get non-uniform mesh refinements, we need to enforce the so-called 1 irregularity rule. The rule doesn't allow for breaking a single element for the second time without breaking adjacent elements first. This is because we do not want