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# The Making of Geometry

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**Abstract**

Geometry has been a source of inspiration in the design of the manmade world for millennia; it also provides representational means enabling development of a concept into a built object. In the past three decades computing methodologies have provided the designer with unprecedented tools to explore highly complex forms, create digital models and fabricate them. This paper describes a computational methodology for the transition of forms from abstract geometric configurations to physical objects: a parametric design process assists from the initial ideation to the final prototyping with 3D printing technologies. The five regular polyhedra are used as a case study; this paper explores how parametric based procedures develop these geometric shapes into digital models of structures to be fabricated in different sizes and materials.


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## 1. Introduction

The role of mathematics in the understanding and interpretation of the physical world is at the foundation of science and perhaps can be summarized in Galileo Galilei's poetic definition of the universe "written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures" [1]. Mathematics underlies laws of nature and it has been used to measure and understand the physical world; nature itself presents geometric arrangements in the majority of forms, found in physics and biology.

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Mathematics and geometry have had a major role in architecture and any manmade fabrication, providing artists, architects and designers with sources of inspiration, representational means and structural verification. Proportions in architecture, symmetry in paintings and the harmonic sequences in the sound of music are only a few of the countless examples of the presence of mathematics in human arts endeavors. The statement from the biologist D'Arcy Thompson "form is a diagram of forces" [2] has guided not only structural engineering explorations but has also inspired and provided aesthetic intention for design production at different scales and functional requirements throughout many historical periods and styles [3].

Despite the entanglement between geometry and nature, somehow the world of geometry is abstracted from the constraints of the physical world ruled by forces and their interactions. Points, lines and surfaces do not have size limitations and are not ruled by the gravity, heat and other physical constraints of the physical products of design. Recent computational digital methodologies can assist in the translation from ideal geometric shapes to digital and physical models of design objects which can be constructed in different materials and sizes.

This paper outlines a computational methodology which can translate geometric entities into digital and physical models using a set of operations in conjunction with user defined parameters. The five regular solids are explored as a case study; the methodology is software independent. Some of the models have been fabricated as stand-alone using 3D printing technologies while others integrate 3D printed parts with off-the shelf components to create a cost effective product.

## 2. The Platonic Solids in Geometry

The choice of the regular convex polyhedra as a case study derives from their relevance not only in geometry but also in art, architecture and design [4]. Each of these solids is composed of equal and regular faces, that is, by regular convex polygons. A regular polyhedron can be identified by the Schläfli symbol {p, q} where p indicates the number of sides of the regular polygon — defining the faces of the polyhedron— and q is the number of polygons meeting at each vertex [5]. For example, according to the notation of the Schläfli symbol, p =3 denotes a triangle, p=4 a square, p=5 a pentagon. In each polyhedron a set of faces meets in a vertex defining a solid angle: all the solid angles of a regular polyhedron are equal. Only five regular convex polyhedra exist, due to the property of their faces and that there is only a limited number of the same regular polygons meeting in one solid angle with sum of the planar angles less than 2p [6].

Another characterizing element for each polyhedron is the dihedral angle —that is the angle between two adjacent faces. Table 1 summarizes the geometric properties for the five solids, which characterize each polyhedron and will be used in later sections to define computational construction rules.

Table 1. The Five Regular Polyhedra as Geometric Entities

| Polyhedron Name | Number of Faces | Number of Edges | Number of Vertices | Schläfli Symbol | Dihedral Angle |
|---|---|---|---|---|---|
| Tetrahedron | 4 | 6 | 4 | {3, 3} | 70.53° |
| Hexahedron (Cube) | 6 | 12 | 8 | {4, 3} | 90° |
| Octahedron | 8 | 12 | 6 | {3, 4} | 109.47° |
| Dodecahedron | 12 | 30 | 20 | {5, 3} | 116.57° |
| Icosahedron | 20 | 30 | 12 | {3, 5} | 138.19° |

The five polyhedra are also called Platonic solids, from Plato who discussed them from his cosmological approach in the dialogue *Timaeus*. Plato assigned the form of each solid to one of the natural elements which were believed to be the fundamental matter of the universe: the tetrahedron was associated to fire, the cube to earth, the octahedron to air, the icosahedron to water, and the dodecahedron was assigned to the ether [7].

The regular solids are characterized by spherical symmetry: from a point on a sphere only five sets of symmetrical points can be individuated, which originate from that point and holding the same distance from the center. The symmetry related properties make the polyhedra of great interest in architectural and product design.