Minerals Engineering 102 (2017) 18–29

Contents lists available at ScienceDirect

Minerals Engineering

journal homepage: www.elsevier.com/locate/mineng

Application of linear circuit analysis in evaluation of mineral processing circuit design under uncertainty

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ARTICLE INFO

Article history:
Received 29 August 2016
Revised 28 November 2016
Accepted 1 December 2016

Keywords:
Circuit design
Uncertainty
Linear circuit analysis
Monte Carlo simulation
Law of propagation of error

ABSTRACT

This paper describes an approach to estimate uncertainty propagation in mineral processing separation circuits. Since most modeling and simulation tools only consider deterministic input values, the inherent uncertainty induced by variable feed grade and stage unit recovery are not included in the design and optimization process. The method described in this paper combines the linear circuit analysis approach and the law of the propagation of errors. The result is a method that can be used to estimate uncertainty propagation, even in the early design stages where extensive experimental data is unavailable. To validate this approach, Monte Carlo simulations were conducted on 35 simple two, three, and four-unit circuit designs and the data was analyzed to show distribution statistics for circuit recovery, product grade, and separation efficiency. Both methods show nearly identical results, but the new methodology also gives fundamental insight on the reasons why certain circuits propagate uncertainty for specific performance indicators.

1. Introduction

1.1. Background

Mineral processing and other physical separation plants often employ complex multi-unit circuits to overcome the inherent inefficiencies of single unit operations. In most cases, single units are deficient in meeting final product specifications while maintaining recovery and grade at economically-favorable levels. Additionally, these single unit inefficiencies also lead to product variable and prediction uncertainty, which is then compounded by varying feed grades and slow lagging process control. Multi-unit circuits can mitigate these issues; however, the circuit design must be carefully considered, as additional units do not necessarily equate to improved performance. As a result, the optimal design of the separation circuit is a complex and open-ended engineering problem that requires a comprehensive and deliberate approach. Unfortunately, the vast number of feasible configurations as well as competing design objectives have led to an overreliance on trial-and-error and iterative approaches based on a deterministic solution (Noble, 2013; Noble and Luttrell, 2014). Uncertainty is typically not considered in the early design stages, even though sales contracts often specify narrow margins on final product specifications and design decisions are often based on the constraints imposed by the contract. Despite the aforementioned problems, a comprehensive solution is essential due to the demands for more profitable and efficient circuits to treat more sophisticated ore resources with diminishing feed grades (Jamett et al., 2012; Montenegro et al., 2013b). Considering the large flow volumes, high capital costs, and relative rigidity of the final flowsheet, considerable effort in the initial testing phase must guarantee that a suitable separation circuit is selected relatively early in the design process (Noble, 2013; Noble and Luttrell, 2014).

Prior to the advent of process modeling and simulation tools, most circuit designs were determined by heuristic methodologies and established industry practices (Lauder and McKee, 1986; Wills, 2011; Lucay et al., 2012). Given the low level of complexity and lack of mathematical rigor, these strategies are easy to implement and, even today, are widely-accepted in the industry. However, since they rely on experiential rules, these classical methodologies may totally fail to achieve an optimal solution (Mendez et al., 2009; Lucay et al., 2012). Over the last 30 years, these heuristic approaches have been slowly displaced by numerical techniques, chiefly phenomenological process modeling and simulation (Lynch et al., 1981; King, 2012). With the advent and development of these techniques, circuit selection has favored a

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http://dx.doi.org/10.1016/j.mineng.2016.12.002
0892-6875/

more rigorous approach since these tools provide a means to assess circuits on the basis of one or more techno-economic performance indicators. Unfortunately, without a prescribed optimization strategy, these tools can lead to large-scale iterative trials, where several circuits are simulated repeatedly until a suitable candidate is identified.

To hasten this approach, numerous researchers have identified optimization approaches that mitigate the need for blind iterative simulation, and a comprehensive review of these strategies has been presented by Mendez et al. (2009). Most of these circuit optimization approaches use a superstructure to represent the set of alternative circuit designs. A mathematical model is then developed, and a search strategy is used to identify the optimal circuit based on one or more specified objective functions. An optimal (or pseudo-optimal) solution may be obtained by utilizing one of several search algorithms, including: genetic search algorithms (Guria et al., 2005, 2006), mixed integer programming (Cisternas et al., 2006; Jamett et al., 2015; Calisaya et al., 2016), integer programming, and other methods (Schena et al., 1996, 1997). Despite the merits of this work, contemporary industry practice still favors classical heuristic solutions. Optimally-derived circuits occasionally produce impractical results, and the optimization approaches are considerably complex. The search algorithms are strongly dependent on the input data and the accuracy of the process models. For example, Cisternas et al. (2014) has shown that the optimal circuit selection is highly dependent on the mineral market price and the specific objective function used in the analysis. Finally, since the solutions are a result of brute mathematical calculations, the optimization approach does not provide any fundamental information on what causes one circuit to be better than another.

As an alternative to the modeling, simulation, and numeric optimization design approach, linear circuit analysis (LCA) is a tool that evaluates and ranks separation circuit designs through simplified algebraic formulations (Meloy, 1983). The method is particularly useful in the preliminary design stages, as it does not rely on a strict process model and thus requires limited *a priori* information on the equipment specifications and feed conditions. During these initial stages, detailed experimental data on circuit performance (such as locked-cycle or pilot tests) can be untimely or costly, and LCA can provide a rigorous and fundamental approach to guide circuit designers to an optimal solution. Nevertheless, LCA cannot completely supplant advanced simulation and circuit optimization techniques, as these approaches are still required for the final solution and performance benchmarking. LCA is fundamentally restricted by the assumption of process linearity, which specifies that a unit's partition function is not influenced by particle-particle interactions. While this assumption is not wholly valid for processing plants, several industrial applications have verified linearity in some cases via experimental investigations (Harris and Cuadros-Paz, 1978; Williams and Meloy, 1983; Williams et al., 1986); and the practical utility of LCA has been repeatedly demonstrated in both greenfield (Luttrell et al., 1998) and operating (McKeon and Luttrell, 2005; McKeon et al., 2012) plant designs.

1.2. Uncertainty consideration in circuit design

Most of the preceding circuit design methodologies, including LCA, have been constructed, implemented, and validated using a deterministic modeling approach. Nevertheless, several researchers have shown that uncertainty can be a significant factor in many mining and mineral processing applications. Kraslawski (1989) extensively reviewed various economic and technical uncertainties in chemical processing operations, including uncertainty in the final product price, feed conditions, and kinetic constants. From a mineral processing perspective, Ghaffari et al. (2012) systematically analyzed mineral assays and flow rates of several streams in a lead-zinc flotation plant. Over the 3 month period of this investigation, the plant data clearly showed the variability of different factors, such as feed grade and flow rate as well as unit recovery. Moreover, the data addressed the dramatic impact that these uncertainties have on the separation performance of the flotation circuit. Other researchers have also evaluated the influence of parameter uncertainty on the variability of flow rates in flotation circuits (Xiao and Vien, 2003) and the variance of SAG mill power draw (Karamoozian et al., 2008). The summative results from these studies show that process uncertainties must be incorporated in the circuit design procedure to achieve an optimal circuit design.

Uncertainty is a broad term, and may be haphazardly applied in the context of mineral processing plant design. Generally, sources of uncertainty in physical operations may include measurement errors, systematic errors, natural variation, inherent randomness and subjective judgments (Morgan et al., 1992; Hoffman and Hammonds, 1994; Regan et al., 2002). Simonsen and Perry (1999) proposed one method of classifying uncertainty in the mining operations by identifying the final parameter where the uncertainty is expressed. This classification identified various uncertainty classes including market price, ore reserve, mining costs, schedule duration, mining methods. Other researchers use a more mechanistic classification, which instead isolates the fundamental cause of the uncertainty as either epistemic (i.e. lack of knowledge) or stochastic/aleatory (i.e. inherent randomness that arises from natural heterogeneity) (Bárdossy and Fodor, 2001; Der Kiureghian and Ditlevsen, 2009; Caers, 2011; Lisitsin et al., 2014; Jamett et al., 2015). Epistemic uncertainty may be reduced by further investigations; however, stochastic uncertainty may not be eliminated or even reduced in many cases. During the design stage, the inherent variability of the final product price is determined by the commodity market and, similar to the feed and ore grade parameters, represents stochastic uncertainty. Alternatively, uncertainty in the unit recovery is an epistemic uncertainty and can be varied by using different design conditions and operational parameters (Jamett et al., 2015).

While individual causes of uncertainty are significant, the circuit designer must also consider the compounded uncertainty imputed by the actual circuit design and stream connections. Current predictive modeling tools often consider static input values; however, some recent studies have investigated the impact of stochastic input data on the final separation performance for various circuit configurations (Lucay et al., 2012; Cisternas et al., 2015; Jamett et al., 2015; Montenegro et al., 2015). Since global optimization through experiments is nearly impossible, Lucay et al. (2015), Sepúlveda et al. (2014) and Lucay et al. (2012) evaluated the significance of each individual unit in determining the variability of global recovery using local and global sensitivity analysis methodologies. Jamett et al. (2012) and Jamett et al. (2015) applied stochastic programming for flotation design using a simple flotation model and analyzed the effect of uncertainty in the circuit design process. More recently, Montenegro et al. (2015) studied the influence of the statistical distributions of input data on the resultant distribution for the global recovery, while Sepúlveda et al. (2014) evaluated the importance of statistical distribution types in addition to the compounded uncertainty inherent to the circuit design. Altogether, these prior studies demonstrate the significance of uncertainty in the circuit design problem and highlight the need for better fundamental understanding on the mechanisms that cause uncertainty propagation in mineral processing circuits.

1.3. Objectives

Given the complexity of the current optimization strategies for mineral processing circuit design, this paper presents a novel