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Experimental study on non-linear behavior of breakage rates due to fines generation in wet batch milling

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ABSTRACT

The mechanism of wet grinding is still an object of interest and the better estimation of grinding kinetics provides greater benefits in terms of energy saving. The primary objective of this paper is to investigate reasons and causative factors contributing to nonlinearities in breakage rates for wet grinding systems. In ball milling the breakage rates vary mainly with the size distribution of the powder generated in the mill. 1st order and 2nd order breakage kinetics have been considered in this work to provide detailed insight into the mechanism of milling during wet grinding operations. Two methods have been used for breakage rate parameter estimation: cumulative input procedure and incremental input procedure. These methods are compared against each other to get a better understanding of how breakage rates evolve. Three different ore types are used as natural and monosized feeds. Results derived from both methods have been lucidly explained. The 2nd order specific selection functions, based on incremental inputs of specific energy, displayed inherent nonlinearities in the wet grinding process. It was observed that the breakage rates varied with the size consist in the mill for all three ore types.

1. Introduction

Comminution has predominantly been the fundamental step in the process of extraction of valuable minerals and metals from ore bodies. Grinding processes and operations, in particular, have always been at the forefront of comminution-related works. Grinding operations are of immense and specific importance to mineral processing and cement industries, since fifty percent of the cost of metal production is incurred in crushing and grinding.

Several attempts at predicting and determining the accuracy of population balance modeling for industrial mill scale-up designs for dry ball mill grinding systems have been made in the recent past (Herbst et al., 1973; Malghan, 1975; Malghan and Fuerstenau, 1976). Correlation of derived selection and breakage functions with mill diameter is the most fundamental step (Malghan, 1975; Malghan and Fuerstenau, 1976).

Population balance modeling for scale-up of ball mills for wet grinding purposes is a relatively new and novel approach. From an industrial perspective, wet grinding is more significant and common than dry grinding. Wet grinding processes encumber the treatment and analysis of inherent and innate nonlinearities that result directly due to the breakage process of particle populations in ball mills (Herbst, 1971; Kim, 1974). A linearized population balance model adept for wet grinding purposes can have its parameters correlated with specific mill operating variables in a metaphoric fashion, quite akin to its correlation with the specific power draft used in dry grinding processes. This, in turn, can help explain the exact consequence of mill design variables on the grinding process and on scale-up.

The primary objective of this paper is to investigate reasons and causative factors contributing to nonlinearities in breakage rates for wet grinding systems. An energy-discretized approach helps in understanding the relationship between particle breakage rates and specific energy input, which, in turn, may prove useful for improving mill efficiency for wet grinding processes. Three different ore bodies have been used to illustrate the variation in breakage rate behavior. Narrow inputs of specific energy in an incremental manner are used to illustrate the ability of the population balance model to predict product particle size distributions. 1st order incremental inputs of specific energy fully illustrates the increase in breakage rates for coarser size fractions of the particle population, with a subsequent decrease in the breakage rates for finer size fractions, as a function of fulcrum position. Here, fulcrum denotes the approximate particle size above which breakage rate increases and below which breakage rate decreases. Detailed description of parameter estimation for predictive simulation has

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been discussed. Effect of variation in percent solids and slurry filling have also been discussed, with close focus on specific energy input and the described fulcrum positions. Comparison of breakage rates during grinding of mono-size material and natural size material has also been discussed. The ores used in this study are limestone ore (softest), quartzite ore (extremely hard and brittle), and gold ore (extremely hard due to presence of granitic and gneissic rock). Particulate environment present in the mill during wet grinding processes has been effectively used to study and describe the nonlinearities in breakage rates.

2. Modeling background

For the last the forty-five years, the population balance model has been used for the analysis and simulation of the batch grinding process. The well-known batch grinding equation is given as:

$$\frac{d[Hm_i(t)]}{dt} = -S_i(t)Hm_i(t) + \sum_{j=1}^{i-1} b_{ij}S_j(t)Hm_j(t) \tag{1}$$

Under the assumption of constant hold-up, H, in the batch mills,

$$\frac{d[m_i(t)]}{dt} = -S_i(t)m_i(t) + \sum_{j=1}^{i-1} b_{ij}S_j(t)m_j(t) \tag{2}$$

In Eq. (1), $m_i(t)$ defines the material mass fraction present in the ith interval at any given time t. $S_i(t)$ characterizes the size-discretized breakage rate function for the size interval i, thereby accounting for the fractional rate of breakage of material from the ith size interval. b_{ij} defines the size-discretized breakage function that denotes the fraction of the product material derived from primary breakage in the jth interval and subsequently found in size interval i (Herbst et al., 1973; Herbst, 1971). Size-discretized breakage rate functions are usually dependent upon particle size distribution in the mill at any arbitrary time t, given as;

$$S_i(t) = S_i(m_k(t)), \quad for\ k = 1, 2, \ldots, n \tag{3}$$

These size-discretized selection functions are dependent on particle size distribution, dependent on time since size distribution varies with time (Herbst et al., 1973; Herbst, 1971; Kim, 1974). A case of linearity of such a kinetic model with constant coefficients is considered valid when the size-discretized selection and breakage functions are individually independent of the particle size distribution in the mill (Herbst, 1971; Kim, 1974).

The size-discretized breakage functions are independent of the particle size distribution within the mill. The breakage functions are assumed to be a material property and not dependent on the milling environment or operating conditions. Hence, this is given by:

$$b_{ij} \neq b_{ij}(H, m_i(t)) \quad for\ i = 1, 2, \ldots, n \tag{4}$$

Further the normalizability assumption is invoked,

$$b_{ij} = b_{i-j+1,1} \tag{5}$$

Under this assumption the breakage function for all sizes $(b_{i2}, b_{i3}, \ldots . b_{i,n-1})$ are obtained directly from b_{i1}. Hence, one needs to estimate b_{i1} only from experiments.

The model Eq. (2) can be transformed into energy normalized PBM (population balance model). Under the assumption the power draw is constant,

$$\overline{E} = Pt/H \tag{6}$$

where P is the power (kW), t is the batch grinding time (min) and H is the ore mass-hold up in the mill (kg).

$$\frac{d[m_i(E)]}{dt} = -S_i^E m_i(\overline{E}) + \sum_{j=1}^{i-1} b_{ij}S_j^E m_j(\overline{E}) \tag{7}$$

The energy normalized Eq. (7) implies that the particle size distribution evolves with the specific energy input (kW h/ton), in conformity with the fundamental laws of grinding such as Bond's or Rittinger's law.

Following the assumption in the linear model formulation of (1) S_i^E could be considered constant, a case often observed in dry grinding of ores. However, in wet grinding S_i^E changes depending on the size distribution within the mill. In all cases, b_{ij}, is considered constant since it is a pure material property.

3. Estimation of breakage rates and breakage distribution

Hereafter, we restrict ourselves to the discussion of breakage rates in the energy normalized PBM Eq. (7). The relationship illustrated in Eq. (7) clearly enunciates the advantage that S_i^E and b_{ij} can be determined directly from milling experiments (Austin et al., 1984; Herbst and Fuerstenau, 1968; Austin and Bhatia, 1971). However, the linear model described in Eq. (7) is valid only for a short duration of grinding time because the breakage rate S_i^E changes with the size distribution of powder in the mill.

For the purpose of estimation of breakage rate functions it is customary to use a functional form for breakage function.

$$B_{ij} = \phi(x_i/x_{j+1})^{\alpha_1} + (1-\phi)(x_i/x_{j+1})^{\alpha_2} \tag{8}$$

Thus the breakage function is described as a function of sieve size ratios. The slopes α_1 and α_2 describes the slopes of the function in the coarser and finer sizes. Likewise, a functional form would be necessary to link the size dependent energy specific breakage rates. Therefore, instead of estimating (n − 1) rate values, it can be reduced to two or three. The functional form adopted in this work is:

$$S_i^E = S_1^E exp\{\xi_1[ln(\bar{x}_i/\bar{x}_1)]\} \tag{9}$$

for first order function and

$$S_i^E = S_1^E exp\{\xi_1[ln(\bar{x}_i/\bar{x}_1)] + \xi_2[ln(\bar{x}_i/\bar{x}_1)]^2\} \tag{10}$$

for the second order function. The first order function is a straight line in the usual logarithmic plot and hence easy to understand the nonlinearities via thus function. Although, slightly more complex, the second order function introduces a parabolic curve in the logarithmic plot.

The solution of the model in Eq. (7) is well known. Therefore, the estimation of unknown parameters $\left(\phi, \alpha_1, \alpha_2, S_1^E, \xi_1, \xi_2\right)$ is accomplished by minimizing the objective function

$$\Omega = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(m_{ij}^{exp} - m_{ij}^{model}\right)^2 \Big/ (mn - p) \tag{11}$$

where the summation is over m grind times and n sizes. The square root of Ω root mean squared value (RMS) has implications in statistics of the linear model fitting. But here we use RMS to simply asses the quality of fit. The denominator denotes the degrees of freedom of the sum of squares in the numerator. Here *p* denotes the number of parameters used to estimate the selection function (2 for first order and 3 for second order).

When a model is fitted, it is imperative that the number of parameters searched for is a few as possible. In batch grinding modeling it is well known that at least the breakage function $(\phi, \alpha_1, \alpha_2)$ can be determined from monosize grinding data. The method is known as "Zero order production of fines" (Herbst and Fuerstenau, 1968).

Briefly stated, the method requires first the estimation of S_1. This is done by plotting the fraction remaining in the top (coarsest