Powder Technology 317 (2017) 6–12

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

# Sensitivity analysis of Austin's scale-up model for tumbling ball mills — Part 2. Effects of full-scale milling parameters

François K. Mulenga *

Department of Electrical and Mining Engineering, University of South Africa, Florida Campus, Private Bag X6, Johannesburg 1710, South Africa

ARTICLE INFO

*Article history:*
Received 1 March 2017
Received in revised form 11 April 2017
Accepted 19 April 2017
Available online 21 April 2017

*Keywords:*
Batch milling
Wet milling
Milling parameters
Scale-up procedure
Residence time distribution

ABSTRACT

The need for scale-up of laboratory milling data to full operation is critical especially for the proper selection of design parameters. This paper investigates the extent to which full-scale milling conditions and design parameters influence the discharged product.
Austin's scale-up procedure for batch grinding data is used to this end and is applied to a continuous mill operated in open circuit. The circuit was simulated in steady-state regime using randomly generated parameters within predefined ranges. These enabled the iterative computation of the corresponding mill products, average characteristic sizes and standard deviations.
Simulation outcomes suggest that mill diameter, top-up ball diameter, in-mill flow pattern, and two scale-up correction factors in Austin's model have a greater bearing on mill product. The two correction factors account for the change in mill diameter and ball size from batch to full-scale milling respectively.


## 1. Introduction

Mulenga [1] investigated the influence of the laboratory-based selection function and breakage function parameters on the quality of the product of a continuous mill in a forerunner to the present work. The scale-up procedure for batch grinding data proposed by Austin et al. [2] was employed to this effect and applied to a full-scale mill in open circuit. In this paper, the focus is on the effects of the technical specifications and operating conditions of the full-scale mill. The properties of the mill feed and key correction factors inherent to Austin's scale-up model are also considered.

Simply put, Austin's scale-up scheme relies on the preparation of representative mono-sized feed samples. Then, a series of laboratory batch grinding tests is carried out to measure the milling characteristics of the material, that is, the breakage function and selection function parameters. The laboratory-based parameters are finally used in the scale-up model to predict the selection function, the breakage function and the overall performance of the full-scale mill under consideration.

In terms of the selection function, the scale-up model can be summarised as follows [2]:

$$S_i(d) = a_T\,(x_i)^{\alpha}\,\frac{1}{1+\left(\frac{x_i}{C_1\mu_T}\right)^{\Lambda}}\,C_2\,C_3\,C_4\,C_5 \tag{1}$$

Where

$$C_1 = \left(\frac{D}{D_T}\right)^{N_2}\left(\frac{d}{d_T}\right)^{N_3} \tag{2}$$

$$C_2 = \left(\frac{d_T}{d}\right)^{N_0} \tag{3}$$

$$C_3 = \begin{cases} \left(\frac{D}{D_T}\right)^{N_1} & , D \le 3.81\ m \\ \left(\frac{3.81}{D_T}\right)^{N_1}\left(\frac{D}{3.81}\right)^{N_1-N_4} & , D > 3.81\ m \end{cases} \tag{4}$$

$$C_4 = \left(\frac{1+6.6J_T^{2.3}}{1+6.6J^{2.3}}\right)\,\exp[-c(U-U_T)] \tag{5}$$

$$C_5 = \left(\frac{\phi_c-0.1}{\phi_{cT}-0.1}\right)\left(\frac{1+\ \exp[15.7\times(\phi_{cT}-0.94)]}{1+\ \exp[15.7\times(\phi_c-0.94)]}\right) \tag{6}$$

With $a_T$, $\mu_T$, $\alpha$, and $\Lambda$ being the laboratory-based selection function parameters whereas $N_0$, $N_1$, $N_2$, $N_3$, $N_4$, and $c$ are the scale-up correction factors.

* Corresponding author.
*E-mail addresses:* mulenfk@unisa.ac.za, mk.francois@yahoo.com.

http://dx.doi.org/10.1016/j.powtec.2017.04.043
0032-5910/

The breakage function, on the other hand, is expressed as follows [2]:

$$B_{i,j} = \Phi_0 \left(\frac{x_j}{x_0}\right)^{-\delta} \left(\frac{x_{i-1}}{x_j}\right)^{\gamma} + \left[1 - \Phi_0 \left(\frac{x_j}{x_0}\right)^{-\delta}\right] \left(\frac{x_{i-1}}{x_j}\right)^{\beta} \quad (7)$$

Where $\beta$, $\gamma$, $\delta$, and $\Phi_0$ are collectively termed breakage function parameters. The term $x_0$ represents the standard particle size which has the value $x_0 = 1$ mm.

Globally, Eqs. (1)–(7) can be used to characterise a full-scale mill in operation once the laboratory-based selection function and breakage function have been measured for the ore. The full-scale milling conditions are: mill diameter $D$, ball diameter $d$, slurry filling $U$, and mill speed $\phi_c$. Similarly, $D_T$, $d_T$, $U_T$, and $\phi_{cT}$ represent the batch grinding conditions used for laboratory testing.

The model underpinning the scale-up scheme suffers from the fact that it was derived largely using data from mills run in batch mode [3–7]. Building from the previous paper [1], the extent of the dependency of Austin's scale-up model on full-scale parameters is now investigated. To this end, an array of random values generated for one full-scale parameter is inputted into the scale-up model while the other parameters are kept constant. The mill product and its corresponding size characteristics, i.e. the 80% and 50% passing sizes, are then calculated. The average value and standard deviation of the two arrays of passing sizes for the random numbers are also determined. The process is repeated for the remaining parameters. Finally, results are compared so as to determine which among the full-scale parameters affect most the predicted product. The threshold deviation was set at $\pm 25\%$ in line with the level of accuracy required of an engineering project at a pre-feasibility stage [8].

## 2. Model of the open milling circuit

There are numerous articles in the literature explaining the development of the breakage model applicable to open milling circuits, e.g. [1,9–12]. They all share in common the fact that they are based on the population balance model, better referred to as size-mass balance model [13]. The model is basically an inventory of material through the mill done per particle size class. It accounts for the fresh feed, the breakage process inside the mill, and the discharge of product. Suffice it to say for the purpose of this article that the model applicable to a full-scale mill in open circuit can be written as follows [2]:

$$p_i = w_i(t) = \sum_{j=1}^{i} d_{i,j}(t)\, f_j \text{ with } n \geq i \geq j \geq 1 \quad (8)$$

Where $f_j = w_j(0)$ is the initial mass fraction of feed in size class $j$ and $w_i(t)$ is the final mass fraction of product in size class $i$. The term $d_{i,j}(t)$ is a transformation matrix describing the breakage of the feed into the discharged product. It is generated using Eq. (10) accounting for the milling process and Eq. (11) describing the material transport through the mill:

$$d_{i,j} = \begin{cases} 0, & i<j \\ e_j, & i=j \\ \sum_{k=j}^{i-1} c_{i,k}\, c_{j,k}\, (e_k - e_i), & i>j \end{cases} \quad (9)$$

With

$$c_{i,j} = \begin{cases} -\sum_{k=i}^{j-1} c_{i,k}\, c_{j,k}, & i<j \\ 1, & i=j \\ \dfrac{1}{\overline{S_i} - \overline{S_j}} \sum_{k=j}^{i-1} \overline{S_k}\, b_{i,k}\, c_{k,j}, & i>j \end{cases} \quad (10)$$

And

$$e_j = \int_0^{+\infty} \exp\left(-\overline{S_j}\, t\right) \varphi(t)\, dt \quad (11)$$

The average selection function $\overline{S_i}$ found in Eqs. (10) and (11) is given by

$$\overline{S}(x_i) = \overline{S_i} = \sum_k m_k\, S_{i,k} \quad (12)$$

The term $S_{i,k}$ in Eq. (12) represents the selection function of particles of size $x_i$ due to grinding balls of diameter $d_k$; it is calculated using Eqs. (1)–(7). The term $m_k$, on the other hand, is the mass fraction of balls of diameter $d_k$ present inside the mill at steady state. For a ball make-up consisting of grinding media of diameter $d_{\max}$, the equilibrium ball size distribution $m_k$ is calculated as follows:

$$m_k = \frac{d_k^{4-\Delta} - d_{k+1}^{4-\Delta}}{d_{\max}^{4-\Delta} - d_{\min}^{4-\Delta}} \quad (13)$$

where $d_{\min}$ is the minimum diameter of ball that can exist in the mill. $d_k$ is the diameter of balls in size class $k$ present inside the mill. The definition of ball size class is such that $d_k > d_{k+1}$ and $d_1 = d_{\max}$. $\Delta$ is a parameter dictating the wear rate model applied to grinding balls.

The term $b_{i,j}$ in Eq. (10) represents the average breakage function experienced by particles of size $x_j$ reporting to size class $i$ due to the mix of balls of different sizes loaded inside the full-scale mill. It is calculated as follows:

$$b_{i,j} = B_{i,j} - B_{i+1,j} \quad (14)$$

Eq. (14) applies when the cumulative breakage function $B_{i,j}$ is the same regardless of ball size $d_k$. In this case then, $B_{i,j}$ is calculated using Eq. (7).

The mill is generally assumed to behave as three perfectly mixed reactors in series: a large one of average residence time $\tau_1$ and two small ones of similar volume and average residence time $\tau_2$ [2,10]. In this instance, the term $\varphi(t)$ in Eq. (11) is then given by

$$\varphi(t) = \frac{\left[-\dfrac{t}{\tau_2} - \left(\dfrac{\tau_1}{\tau_1 - \tau_2}\right)\right] \exp\left(-\dfrac{t}{\tau_2}\right) + \left(\dfrac{\tau_1}{\tau_1 - \tau_2}\right) \exp\left(-\dfrac{t}{\tau_1}\right)}{\tau_1 - \tau_2} \quad (15)$$

The flow pattern of material represented by Eq. (15) is closely related to the mass of slurry present in the full-scale mill. This then conditions the powder filling $U$ needed in Eq. (5) that can be expressed as follows [14]:

$$U = \begin{cases} \left(\dfrac{J_0}{J}\right) \sqrt{\dfrac{F}{F_1}}, & \text{for } F > F_1 \\ \dfrac{J_0}{J}, & \text{for } F \leq F_1 \end{cases} \quad (16)$$

With $F_1 = 0.63\, \rho_o\, \varphi_c\, D^{3.5}\, \left(\frac{L}{D}\right)$.

Here, $J_0$ is the ball filling corresponding to the static bed of balls reaching the trunnion overflow level; its default value is $J_0 = 34.3\%$