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A synchronized statistical characterization of size dependence and random variation of breakage strength of individual brittle particles

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ABSTRACT

This work directly applies the weakest link formulation of cumulative failure probability for synchronized analysis of both size dependence and random variation of particle strength. This approach is justified from two aspects: First, the breakage of brittle particles belongs to brittle fracture, which is commonly described by the weakest-link postulate. Second, the weakest link postulate allows considering different statistical distributions in addition to the Weibull distribution. Eight sets of published breakage strength dada of different sized glass spheres and irregularly shaped grains made of different materials are proven to fall onto a master curve dictated by the weakest link statistics model.

1. Introduction

Particle comminution and powder granulation refers to the size reduction of a granular product, either as the result of an intentional effort or as an undesirable consequence. This is a topic of significant interest in chemical, pharmaceutical, food, mining, and many other industries [1–6]. Breakage and fragmentation of individual particles is an elementary physical event in the processes of particle comminution and powder granulation. This makes the characterization of breakage strength of an individual particle, often known in brief as the particle strength, a basic requirement in the evaluation of particle size reduction. At a high level, particle strength characterization involves two interrelated aspects: (1). Definition and experimental determination of particle strength; (2). Statistical distribution of particle strength.

- Definition and experimental determination of particle strength. The definition and experimental determination of particle strength lays the foundation for evaluating the behavior of particle breakage. Depending on the specific applications, the breakage process of a single particle has been investigated in either quasi-static or impact loading mode. In an impact test, the variation of breakage mode and size distribution of fragmentation with impact speed is of major interest. A threshold impact speed for the occurrence of breakage is usually determined as an index of the dynamic resistance to breakage for specific particle properties in terms of particle size and mineralogy. The breakage behavior of an individual particle at static loading condition is usually studied on a symmetrical compression setup with the axially symmetrical compressive forces F applied on a spherical particle of radius R via a pair of parallel platens (Fig. 1a), where a is the contact radius. For an elastically isotropic, homogeneous, perfectly spherical particle subject to symmetrically compressive loading, analytical solutions to the stress distributions inside the particle have been developed based on the Hertzian contact mechanics [7,8]. In principle, the stress distribution in such an idealized spherical particle in a spherical-polar coordinate system (r, θ, ϕ), follows

$$\sigma_{ij} = \frac{F}{\pi R^2} g_{ij}\left(\nu, \frac{a}{R}, \frac{r}{R}, \theta\right) = \sigma_{nom} g_{ij}\left(\nu, \frac{a}{R}, \frac{r}{R}, \theta\right)(i, j = r, \theta, \phi) \tag{1}$$

where $\sigma_{ij}(i,j=r,\theta,\phi)$ denotes stress components in different directions (Fig. 1b), g_{ij} is a dimensionless function dependent on Poisson's ratio ν, the ratio of contact radius to particle radius a/R, the normalized radial distance r/R from the center of the particle, and the polar angle θ, $\sigma_{nom}=F/(\pi R^2)$ is the nominal stress. The breakage strength of an individual particle, denoted as $\sigma_{b,}$ is defined as below according to the nominal stress σ_{nom} at breakage,

$$\sigma_b = k\sigma_{nom} = \frac{k \cdot F}{\pi R^2} \tag{2}$$

where $k>0$ is a coefficient dependent on the elastic properties of particles, the mechanical properties of the platens as loading fixtures, and specific loading configurations. The uniaxial compression test is also

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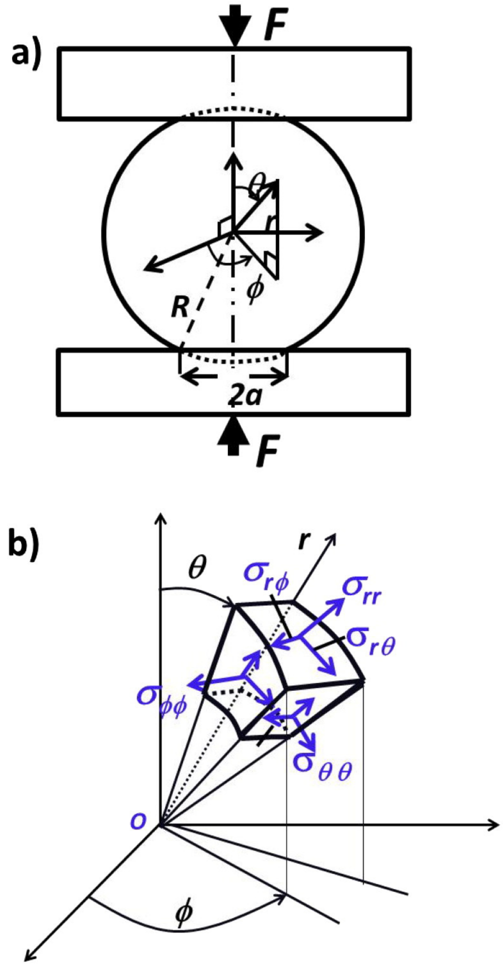

Fig. 1. Axially symmetric compression test setup of a brittle particle (a), and stress components in a spherical-polar coordinate system (r, θ, ϕ) (b).

commonly used to investigate the breakage behavior of non-spherical particles with either well-defined geometries [9] or much irregular shapes [2–6,10,11], with Eq. (2) being adopted to measure the breakage strength in analogy. Usually, the geometrical irregularity of a particle is characterized in terms of sphericity and roundness. According to Wadell [12], sphericity is the ratio of the surface area of a sphere to the surface area of a particle with the same volume, while roundness is the ratio of the average curvature radius of particle asperities to the curvature radius of the largest circumscribed sphere. To highlight the difficulty in accurate stress analysis, Fig. 2 illustrates an irregularly-shaped particle in compression, with the largest circumscribed sphere showing in red dashed line. It is obvious that the coefficient k in Eq. (2) depends heavily on the irregularity of particle shape. Therefore, the breakage strength of a particle is usually directly measured with the nominal stress σ_{nom} by taking $k=1$ in Eq. (2).

- Statistical distribution of particle strength. The breakage of a brittle particle possesses the common features of brittle fracture, namely the inherent scatter of strength and the size effect of strength. The scatter of strength is meant by the inherent variation of breakage strength values measured from a population of particles made of nominally same material with nominally same geometrical shape and size under the same compressive loading condition, due to the random distribution of microdefects residing in a particle in terms of their spatial location, orientation, size and shape. This inherent variation makes the statistical characterization of particle strength as the method of choice for particle breakage assessment. The size effect denotes the dependence of particle strength on both the shape and the size of a particle. For a spherical particle, it is feasible to partition the inherent variation of particle strength and the size effect. However, for irregularly-shaped particles, this becomes impractical. Nevertheless, due to the lack of alternative means, statistical characterization of the breakage strength of irregularly-shaped particles has become an indispensable tool. Predominately, the distribution of particle strength has been described by the two- or three- parameter Weibull statistics as below [13]:

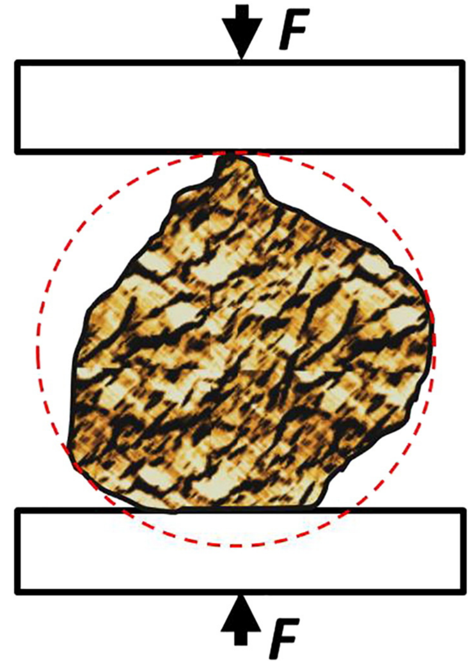

Fig. 2. Uniaxial compression test of an irregularly-shaped particle. The red circle in dashed line depicts the circumscribed circle. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$P=1-\ exp\left[-\left(\frac{\sigma_b-\sigma_{th}}{\sigma_0}\right)^m\right]\ \ (0\leq\sigma_{th}\leq\sigma<\infty) \tag{3}$$

$$\text{or } LnLn[1/(1-P)]=mLn(\sigma_b-\sigma_{th})-mLn(\sigma_0)\ \ (0\leq\sigma_{th}\leq\sigma<\infty) \tag{4}$$

where P is the cumulative probability of failure of a particle, σ_{th} is the threshold strength, σ_0 is the scale parameter, and m is Weibull modulus.

Despite its popularity, the resultant Weibull parameters (m and σ_0) often exhibit strong dependence on particle size [4,6,14,15], in addition to the lack of a rationale for the calibration of a three-parameter Weibull model. One the one hand, the size dependence of m and σ_0 turns Weibull statistics into a good tool only for statistical description of a given sized particle but without prediction capability for other different sized particles. One the other hand, since Weibull parameters relates to the statistical distribution of microdefects in a particle, the size effect on m and σ_0 seems to suggest significant differences in the statistical distribution of microdefects in different sized particles, which is debatable given that the same material and same manufacturing process conditions are adopted for different sized particles.

The weakest link postulate has been commonly adopted in the development of statistical approaches to brittle fracture. It allows taking into account different types of statistical distributions including Weibull statistics. This work directly resorts to the weakest-link formulation for the cumulative failure probability to explore an approach to statistical description and prediction of particle strength distribution.