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Asymptotic solutions for laminar flow based on blood circulation through a uniformlyporous channel with retractable walls and an applied transverse magnetic field

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ABSTRACT

This paper is concerned with asymptotic solutions of a nonlinear boundary value problem (BVP), which arises in a study of laminar flow in a uniformly porous channel with retractable walls and an applied transverse magnetic field. For different ranges of the control parameters (i.e. α, Re and M) arising in the BVP, four cases are considered using different singular perturbation methods. For the first case, unlike those in the existing literature, we make use of the Lighthill method and successfully construct an asymptotic solution with high-order derivatives at the center of the channel. For the second case, under large suction we consider $M^2 = O(1)$ and $M^2 = O(Re)$, respectively, which will further extend the applying range of asymptotic solutions. In other cases, asymptotic solutions with a boundary layer are successfully constructed. In addition, numerical solutions presented for each case agree well with asymptotic solutions, which illustrates that the asymptotic solutions constructed in this paper are more reliable. Finally, the influences of some parameters on flow field are discussed to develop a better understanding of the flow problem.

1. Introduction

Blood circulating in the blood vessel has a strong effect on the human body and also serves as one of the basic substances constituting the human body. Its dynamics is closely associated with people's health. For example, as said by Srivastava [1], atherosclerosis, a leading cause of death in many countries, is one of the phenomenon in which the flow behaviour of the blood in the vessel will be influenced by the intimal thickening of stenos artery. When severe stenosis suppresses the speed of blood, the blood supply and oxygen to the brain are reduced. Under this situation some cells in the brain start to die and then the resulting serious diseases will appear (e.g. strokes). So studies of fluid transport in the vessel can serve to better understand the functions of biological organisms (e.g. lung and cardiac).

When concerning systemic circulation in blood circulation, the blood in the left ventricle is being forced into the aorta by systole and the mitral valve between left ventricle and left atrium is closed. At this juncture the left ventricle forms a vessel with one end closed. Meanwhile, the mass transfer of the vessel between inside and outside can be achieved by the seepage across permeable wall of the vessel [2–4]. Furthermore, some idealized mathematical models are proposed which consider the vessel to be permeable [5,6]. So studies on such flow dynamics can be meaningful in the field of bioengineering and medicine. In 1990, a mathematical model on the viscous flow of Newtonian fluid inside a permeable tube with expanding or contracting cross section was established by Goto and Uchida [7]. In their work, a expansion ratio α and a cross-flow Reynolds number Re (defined in Section 2) were introduced to measure the expansion of the pipe and the mass transfer, respectively. Later, Dauenhauer and Majdalani [8] considered the case that laminar flow in a porous channel with expanding or contracting walls and thus established a mathematical model. So far there have existed some studies on the mathematical model. To list a few, one may count Majdalani et al. [9], Asghar et al. [10] and Hang Xu et al. [11]. On the other hand, some of medical literature have also shown that certain external factors can change the hydrodynamic in blood flow. When the blood is regarded as an electrically conducting fluid, the control of blood flow can be achieved by the application of the magnetic field (Noting that the fluid is often called as Magnetohydrodynamics or MHD). Based on the experimental investigation, Karmilov [12] has revealed that the

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magnetic field exerted a most significant influence on the vascular system. Subsequently, Sambasiva [13] also studied an unsteady MHD blood flow through a porous channel with porous walls. So far, some valuable results on MHD in a pipe have been reported. For example, as said in [14], the effects of MHD on blood flow are as follows: i) to reduce the high shear stress caused by stenosis and hence to prevent the damage to the red and endothelial cells, which will help bioengineers in the design of artificial organs and the treatment of vascular diseases (e.g. [15–18]). ii) to delay the transition from laminar to turbulent flow inside the blood vessel and thus reducing high intensity shear zones, which are unfavorable to the blood and arterial wall. This may be vital to watch out for the symptoms of a carotid artery blockage (e.g. [19,20]). Motivated by above works, we have realized the importance of magnetic field appearing in a model of laminar flow in a porous pipe with expanding or contracting walls. However, very little is known so far about the result of laminar flow in a porous channel with expanding or contracting walls and an applied transverse magnetic field. Therefore, based on the work [8], a principle objective of the current study is to overcome a deficiency in their model that does not account for the presence of a magnetic field. In fact, the investigation of the steady flow of an electrically conducting viscous fluid through a semi-infinite flat plate with an applied transverse magnetic field has been initiated by Suryaprakasrao [21], who obtained an asymptotic solution for small Hartmann numbers (defined in Section 2). Later, Terrill and Shrestha [22,23] extended Suryaprakasrao's work by considering laminar flow in a porous channel with motionless walls and an applied transverse magnetic field. In their studies, based on either numerical or asymptotic approaches, some solutions were obtained for both small and large Reynolds numbers and all values of Hartmann number.

In fact, for the viscous flow in a porous channel with stationary walls, the earliest researcher can be traced back to Berman [24]. In his study, a nonlinear boundary value problem (BVP) with a cross-flow Reynolds number *Re* was obtained from the classical Navier-Stokes equations. For small *Re*, he constructed an asymptotic solution using a regular perturbation method. Subsequently, a number of further studies about the existence of multiple solutions of such a BVP followed shortly thereafter. Among these are the works of Robinson [25], Skalak and Wang [26], Shih [27], Stephen [28] and Lu [29–33]. Recently, when the walls of the channel were not motionless, Hang Xu et al. [11] obtained three solutions for large suction using homotopy analysis method (HAM). In addition, the temporal and spatial stabilities have also considerable attention in the past due to the existence of multiple solutions of the BVP, where one may count Brady [34], Durlofsky and Brady [35], Sobey and Drazin [36], Zaturska, Drazin and Banks [37].

The purpose of this paper is to extend previous investigations by presenting asymptotic solutions for laminar flow in a porous channel with expanding or contracting walls and an applied transverse magnetic field. Specifically, in Section 2, by introducing the flow geometry, governing equations with boundary conditions and a stream function, a BVP (i.e. Eqs. (13)–(14)) including three parameters (i.e. α, Re and M) is obtained. In general, when constructing a perturbation solution of the BVP, we should consider the order of magnitude among these parameters, otherwise the perturbation solution constructed is only valid for the limited scope of parameters. Therefore, Section 3 serves to present asymptotic solutions for different cases. The asymptotic and numerical solutions are compared and discussed in Section 4. Finally, Section 5 concludes the paper.

2. Mathematical formulation of the problem

We assume that the channel is of semi-infinite length with one closed end. In addition, to consider a two-dimensional flow, we assume that the distance $2a$ between the porous walls is much smaller than the channel's width. Both sidewalls are assumed to have equal permeability $-v_w$ and to expand or contract uniformly by a time-dependent rate $\dot{a}(t)$. As shown in Fig. 1, x and y indicate the streamwise direction and the normal direction, respectively. u and v denote the velocity components along $x-$ and $y-$axes. The flow velocity is zero at the closed end ($x = 0$). As a result, the motion of a viscous incompressible and electrically conducting fluid through a porous channel with an applied transverse magnetic field can be described by the following equations:

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

and

$$\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{V} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{V} + \frac{1}{\rho}\boldsymbol{J} \times \boldsymbol{B}, \tag{2}$$

where $\boldsymbol{J}$ and $\boldsymbol{B}$ are given by the Maxwell equations

$$\nabla \times \mathbf{H} = 4\pi \boldsymbol{J}, \tag{3}$$

$$\nabla \times \mathbf{E} = 0, \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{5}$$

and Ohm's law

$$\boldsymbol{J} = \sigma[\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B}], \tag{6}$$

where $\boldsymbol{B} = \mu_m \boldsymbol{H}$, $\mathbf{V} = (u, v)$ and the symbols ν, σ, and μ_m represent the viscosity of the fluid, the electrical conductivity and the magnetic permeability, respectively.

For simplicity, we further assume that a constant magnetic field of strength H_0 is applied perpendicular to the walls and there is no external electric field. Meanwhile, here the induced magnetic and electric fields produced by the motion of the electrically conducting fluid are neglected. With these assumptions the magnetic term $\mathbf{J} \times \mathbf{B}$ in Eq. (2) reduces to

$$\mathbf{J} \times \mathbf{B} = -\sigma H_0^2 \mathbf{V}. \tag{7}$$

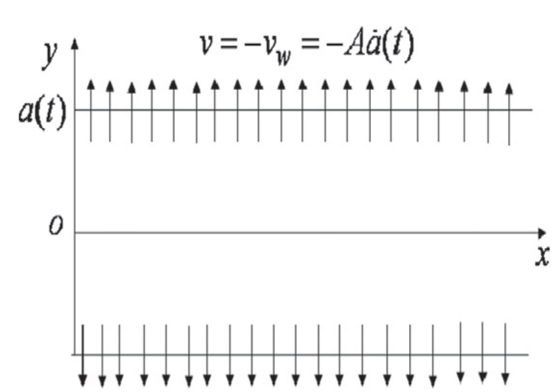

Fig. 1. Physical configuration.