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Dynamic of one and two elliptical particles settling in oscillatory flow: Period bifurcation and resonance state

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ABSTRACT

The inertial migrations of one and two elliptical particles in a two-dimensional channel under oscillatory pressure driven flow are investigated by a finite element arbitrary Lagrangian-Eulerian (ALE) method. The effects of oscillation frequency on rotation behavior, vortex structure, pressure distribution and relative displacement during sedimentation have been studied. The results show that the oscillation can accelerate the eddy formation and shedding behind the particle, and cause a strong modification of the turning couples on the ellipse with lateral displacement. As the frequency increases, the initial 'zigzag migration' at the channel side goes through a period bifurcation transition to the resonance state with 'anomalous rotation' near the wall, and further induces 'rotation shift' near the channel center. The angular velocity increases before the resonance state, while the average vertical velocity decreases with the rising frequency. Moreover, the hydrodynamic interaction between the particles has a close association with the oscillatory effect. In the period bifurcation regions, the higher the oscillation frequency, the bigger the horizontal distance of the opposite rotation particles, and the particles separate fastest at the resonance state. In contrast, an attractor is formed between the pairs in the high frequency, and the two particles are exchanging the lead as settling forward. The final configuration of the pairs is shown to be caused by the Magnus type of lift balancing the wall repulsion and the interplay between particles, and is sensitive to the dynamic drag statistically corresponding to the oscillation in the wake and the periodic discharge of vorticity.

1. Introduction

The migration of particles suspended in fluids exists in many practical utilizations, i.e. coal-water slurry transport, pharmaceutical processing and environmental waste treatment, etc. In most cases, modeling the particle-fluid interaction is difficult because of the complexity of the solid shape and the variety of the flow field. For the transient phenomenon of oscillatory shear, sudden acceleration and deceleration of the flow, the microstructure around the particle can be induced by flow, which generally has a great effect on the apparent motion of the particle. The investigation of the dynamic behavior of suspensions in oscillatory flow has captured much attention in recent years.

The pioneering experiments of Segré and Silberberg [1] have been conducted on the inertial migration phenomenon in a tube flow with Newtonian fluid. They observed that the non-interacting, neutrally buoyant sphere keeps moving along both the wall and the centerline, forming an equilibrium layer at about 0.6 times the radius at a moderately high Reynolds number. Then Feng et al. [2,3] further demonstrated that the dynamic behaviors of non-neutrally buoyant particles could be classified into different flow regimes as the density of the particle increases. Their results indicated that the particle would either oscillate or tumble in rotation due to the periodic shedding of vortices at relative high particle density intervals. Meanwhile, typical numerical work suggested that the driving forces of laterally movement could be identified as an inertial lift due to shear slip [4], a wall repulsion related to lubrication [5], a lift caused by rotation [6] and, in the case of Poiseuille flow, a lift associated with the velocity profile curvature [3,7]. It is confirmed that the particle rotation is determined by the shear stress on its surface resulting from the competition of the density ratio, the initial state, the type of the flow, the fluid rheology as well as the particle shape [8]. The unevenness of local pressure distribution and the corresponding drag-lift correlation give rise to the alternate eddy formation and shedding behind the body, eventually leading to diverse migration trajectories [9,10].

Much of the research in the last two decades has also addressed the elliptical particles. Huang et al. [11,12] analyzed the distributions of pressure and shear stress, as well as the turning couples on the surface of an elliptical particle. They found that the high pressure at a front stagnation point is the stabilizing mechanism which turns the ellipse broadside horizontal. In addition, Qi et al. [13] explored neutrally elliptical particles migrate in Poiseuille flow, and pointed out that the Segre-Silberberg effect also exists in the non-spherical particle and multi-particle systems due to the balance between the inertia effect and wall

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effect. Afterwards, the orientation of the elliptical particle has been investigated by several authors [14,15]. It is concluded that the amplitude and period of the oscillatory motion for an ellipsoid with inertia depend on its initial orientation, and in the near-wall region, ellipsoidal particles tend to align with the mean flow direction. Particularly, Huang et al. [16] studied the rotation behavior of neutrally buoyant spheroidal particles in shear flows, and found that both the prolate spheroid and the oblate spheroid would reach log rolling, inclined rolling and different steady states at different particle Reynolds numbers. Moreover, Xia et al. [17] and Huang et al. [18] focused on the effects of boundaries on the settling behaviors of two-dimensional (2D) and 3D elliptical cylinders, respectively. They found that the particle undergoes a series of transitions in the pattern of settling including oscillation, tumbling, horizontal sedimentation, vertical sedimentation and an inclined mode of sedimentation at low channel blockage ratios. Some of the behaviors in their researches agree qualitatively with our numerical results, and the 'anomalous rotation' of the elliptical particle is certainly observed along the wall in the present paper.

For particle-particle interactions, Fortes et al. [19] performed the drafting-kissing-tumbling (DKT) process for two spherical particles falling against gravity in their experiment with a channel filled with a Newtonian fluid for the first time. Later on, the similar results have been obtained from the numerical simulations [2,20], and they suggested that the trailing particle sucked into the wake of the other only comes out at moderately high particle Reynolds numbers. Alternatively, Aidun et al. [21] presented a cascade of period-doubling bifurcation to a chaotic state by a low dimensional chaotic attractor at relative low particle Reynolds numbers without vortex shedding. In particular, in a rotating flow, it is observed that a pair of spherical particles motion near circular in an out-of-phase manner with regard to their radial positions resemble bicycle pedaling [22]. Furthermore, theoretical investigation of hydrodynamic interactions between two particles [23,24] indicated that the given static friction coefficient and contact separation distance have a linear influence on the transition from a pattern to another. Therefore, the reason behind the eventual attainment of these various states is pointed out to be the interplay between the particle–particle and particle–wall forces, by considering the difference in size and initial position of the particles [25,26]. Indeed, Luo et al. [27,28] investigated the rotation behaviors of one and two particles settling near a vertical wall, and observed that the particle may be 'rotation shift' or 'zigzag migration', depending on initial position. Wang et al. [29] explored the settling of two particles with different initial longitudinal distances and diameter ratios. They manifested that the two particles never undergo the DKT process when the initial distance beyond a certain threshold and the two particles with different sizes are easier to separate than two identical ones.

Wall-bounded oscillation is the actual configuration of flow field change in many different practical situations (i.e. fluid pumping, respiratory system of living beings, etc.). Konstantinidis et al. [30] suggested that the fluctuating velocity fields would modify the wake structure behind the cylinder, and thus to disturb the drag and lift on the particle [31]. The rotation of an inertial spheroid in oscillating shear flow could be chaotic dynamics and the long-term behavior of the particle is unpredictable [32]. Through experimental method, Butler et al. [33] noted that the amplitude of oscillation affects the direction of particle motion, and observed that the particle preferentially moves away from the wall and to the channel center when the oscillation amplitude is large enough. Recently, Sun et al. [34] using the direct-forcing fictitious domain (DF/FD) method presented that the oscillatory Poiseuille flow gives rise to the circular particle closer to the centerline, and emphasized that the oscillation frequency of the pressure gradient affects its equilibrium position significantly. Moreover, by a theoretical approach, Yapici et al. [35] also reported that the particle migrates to the channel center for oscillations with large amplitudes but to the off-center positions or to the wall when the amplitudes are small. In addition, our previous research concluded that there exists an equilibrium position in the high frequency flow for a narrow channel, and the position of the heavier particle is closer to the centerline [36]. However, the transition processes of particle dynamics at different oscillation frequencies are undefined in their studies and little research is available on the particle-particle interaction in oscillatory flows.

In this work, we study the settling of one and two elliptical particles subjected to the pressure driven flow with a finite element arbitrary Lagrangian-Eulerian method, focusing on the effect of oscillation frequency of the flow field on performance of particles. The rest of the present paper is organized as follows. In Section 2, the governing equations describing the motion of both the fluid and the particle are laid down, and the finite element numerical method is briefly introduced. In Section 3, the numerical method is calibrated and validated with the Lattice Boltzmann result. Then, the rotation behavior, the flow microstructure as well as the terminal velocity of an elliptical particle settling at different frequencies are presented in Section 4, and the relative displacement and the hydrodynamic interaction of two elliptical particles sedimentation when they are initially located side by side are shown in Section 5. Finally, in Section 6, conclusions are drawn from the present study.

2. Numerical method

2.1. Governing equations

The governing equations for the fluid domain can be stated as follows:

$$\nabla \cdot \mathrm{u} = 0, \tag{1}$$

$$\rho_\mathrm{f}\left(\frac{\partial \mathbf{u}}{\partial \mathrm{t}} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = \rho_\mathrm{f}\mathbf{f} + \nabla\cdot\sigma, \tag{2}$$

where $\mathbf{u} = (u, v)$, $\mathbf{f}$ and σ are the fluid velocity vector, the body force, and the stress tensor, respectively. For a Newtonian fluid, the stress tensor is defined by the simple constitutive relation.

$$\sigma = -p\boldsymbol{I} + \eta_\mathrm{f}\left[\nabla\mathbf{u} + \nabla\mathbf{u}^\mathrm{T}\right], \tag{3}$$

where p is the pressure, $\boldsymbol{I}$ is the unity tensor, and η_f is the viscosity of the Newtonian fluid.

The rigid particles satisfy Newton's second law for the translational motion,

$$m_i\frac{\mathrm{d}\boldsymbol{U}_i}{\mathrm{d}t} = \mathbf{G}_i + \mathbf{F}_i = \mathbf{G}_i - \int_{\partial\Omega_i(t)} \sigma\cdot\mathbf{n}\,\mathrm{d}S, \tag{4}$$

and the Euler equations for the rotation,

$$\mathbf{J}_i\frac{\mathrm{d}\boldsymbol{\omega}_i}{\mathrm{d}t} = \mathbf{T}_i = -\int_{\partial\Omega_i(t)} (\mathbf{x}-\mathbf{X}_i)\times(\sigma\cdot\mathbf{n})\,\mathrm{d}S, \tag{5}$$

where the index i $(=1, 2)$ represents different elliptical particles; $\boldsymbol{U}_i = (U_{xi}, U_{yi}) = \mathrm{d}\mathbf{X}_i/\mathrm{d}t$ and $\omega_i = \mathrm{d}\Theta_i/\mathrm{d}t$ are the translational velocity and angular velocity of the particle, respectively. m_i Mand $\mathbf{J}_i$ represent the mass and the inertia moment matrix; while $\mathbf{G}_i$ is the body force external gravity field, which includes gravity and buoyancy. $\mathbf{F}_i$ and $\mathbf{T}_i$ are the total force and torque acting on the particle by the surrounding fluid, with S and $\mathbf{n}$ being the surface of the particle and the unit normal vector pointing outward into the fluid, respectively.

2.2. Particle collision

A supplementary collision model is applied to prevent the particles from interpenetrating each other. By following the spring type short-range repulsive force [37], we have adjusted it slightly to account for