Powder Technology xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Compressive sensing for particle size retrieval by using a digital micro-mirror device-based detector

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ARTICLE INFO

Article history:
Received 16 November 2015
Received in revised form 18 May 2016
Accepted 10 August 2016
Available online xxxx

Keywords:
Particle sizing
Scattering measurements
Compressive sensing

ABSTRACT

In this paper, compressive sensing (CS) was implemented to reconstruct the particle sizes in the particle sizing instrument with a digital micro-mirror device (DMD)-based detector. Different size distributions were used to compare the performance of the proposed sampling method. ℓ_1-norm-based reconstruction algorithm was implemented to retrieve the size distribution of particles. In the simulations, data contaminated by different additive noises were applied to size retrieval at several sampling rates. Experiment on a phantom was carried out in this work. The results showed that the proposed method based on CS was robust to the noises and efficient as less samplings were required.

1. Introduction

How to retrieve the size distribution of opaque particle clusters has attracted intensive attention from researchers as well as engineers in many applications. There already exist a variety of particle sizing methods, such as sedimentation, sieving, electric sensing, light scattering techniques, and image analysis and etc. Among all these approaches, light scattering techniques have been broadly investigated and implemented for their high speed, high accuracy, non-invasiveness, and wide dynamic range [1]. Usually, the tiny particles can be treated as small spherical balls, and the Lorenz-Mie theory gives a precise description of the optical field when a collimated light beam is scattered by spherical particles [2,3]. However, the formulae of the theory are very complicated in general cases. An effective simplification of the theory, namely, Fraunhofer diffraction, can be achieved within the near-forward angles, especially for particles that are significantly larger than the wavelength of the laser in use [4]. As a result, a variety of the so-called Fraunhofer diffraction particle-sizing instruments have been promoted in both laboratory and industries. Typically, concentric photosensitive arc-shaped detectors are used to detect the angular distribution of the light energy/intensity in these instruments [5]. However, there are two shortages which distort the precision and reduce the convenience of these instruments. One is non-uniformity of the responsivities of the photosensitive arc-shaped detectors to light intensity [6], and the other is misalignment of the photoelectric detector [7]. To solve these two problems, single pixel camera [8] structure was applied in the particle sizing instrument by using a digital micro-mirror device (DMD)-based detector [9]. The DMD used in the particle sizing instrument provides a flexible configuration to detect the angular distribution of the light energy. Unlike the typical separated concentric photosensitive detectors, the arc-shaped mirror arrays in the DMD can be overlapped with each other, and this configuration achieves better performance than the typical one [9].

In this paper, Compressive sensing (CS) has been implemented to reconstruct the particle size distribution from the samplings of the light energy distribution of the diffraction pattern. Each sampling was obtained from the random distributed concentric arcs, which were constructed based on the observation matrix in CS theory. In the simulations, different levels of additive noises were used to compare the performance of the proposed method for three typical distributions. ℓ_1-norm-based reconstruction algorithm was implemented to retrieve the size distribution of particles. In the experiments, a phantom was used as the sample. The results showed that the proposed method was effective and robust for particle sizing from noise-contaminated data.

2. Particle size retrieval via compressive sensing

If a collimated light beam is scattered by spherical opaque particles, whose diameters are far larger than the wavelength of the incident light, i.e. λ, the diffracted light energy can be collected by using N concentric arc-shaped detectors. For simplicity, it is assumed that M different diameters of particles exist in the samples. For the m-th

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http://dx.doi.org/10.1016/j.powtec.2016.08.020
0032-5910/

($1 \le m \le M$) particle diameter, if there are q_m particles of the same size d_m, the energy captured by the n-th detector can be expressed as follows

$$e_n = \frac{3I_0}{4\rho}\sum_{m=1}^{M}\frac{w_m}{d_m}\left[J_0^2\left(\frac{\pi d_m}{\lambda}i_n\right)+J_1^2\left(\frac{\pi d_m}{\lambda}i_n\right)-J_0^2\left(\frac{\pi d_m}{\lambda}o_n\right)-J_1^2\left(\frac{\pi d_m}{\lambda}o_n\right)\right], \quad (1)$$

here I_0 is the intensity of the incident light, $J_1(\bullet)$ is the Bessel function of the first order, i_n and o_n are diffraction angles respectively related to the inner and outer radiuses of the n-th arc-shaped detector. ρ is the mass density of particles, $w_m = q_m\rho\pi d_m^3/6$ is the weight of the particles with diameter d_m, $J_0(\bullet)$ is the Bessel function of zero order. Then energy of the diffracted light onto each arc can be calculated by

$$\boldsymbol{E} = \boldsymbol{TW}, \quad (2)$$

where

$$\boldsymbol{E} = (e_1, e_2, \cdots, e_N)^T, \quad (3)$$

$$\boldsymbol{W} = (w_1, w_2, \cdots, w_M)^T, \quad (4)$$

$$\boldsymbol{T} = \begin{pmatrix} t_{1,1} & t_{1,2} & \cdots & t_{1,M} \\ t_{2,1} & t_{2,2} & \cdots & t_{2,M} \\ \vdots & \vdots & \vdots & \vdots \\ t_{N,1} & t_{N,2} & \cdots & t_{N,M} \end{pmatrix}, \quad (5)$$

and

$$t_{n,m} = \frac{1}{d_m}\left[J_0^2\left(\frac{\pi d_m}{\lambda}i_n\right)+J_1^2\left(\frac{\pi d_m}{\lambda}i_n\right)-J_0^2\left(\frac{\pi d_m}{\lambda}o_n\right)-J_1^2\left(\frac{\pi d_m}{\lambda}o_n\right)\right]. \quad (6)$$

Here, if the light energy distribution coefficient matrix, i.e. **T**, is predetermined, the particle size distribution, i.e. **W**, can be retrieved by solving Eq. (2) from the measured data **E** through proper reconstruction algorithms, such as Chahine regularization algorithm, Tikhonov regularization algorithm, ℓ_1-norm-based algorithm, modified Landweber algorithm, etc. [10–13].

Compressive sensing is considered as a robust and efficient sampling method for signal reconstruction [14,15]. In this work, the signals of light energies on the detectors, i.e. **E**, can be treated as sparse in the space denoted by the observation matrix in CS theory, then the sampling of the signals can be expressed as

$$\boldsymbol{y} = \Phi\boldsymbol{E}, \quad (7)$$

where **y** is M times samples of **E** by changing the DMD configurations, Φ is the observation matrix, to denote the sampling s of the signal **E**.

In this paper, the diffraction pattern is divided into concentric arcs with equal width on DMD. Each arc of the DMD reflects a certain part of the diffraction pattern, and the reflected light energy will be detected by a single-point photodiode. If the diffraction pattern is divided into N parts, the light energy distribution of diffraction pattern can be denoted by an N-length vector, **E**. Each row of the sensing matrix Φ is one sampling of the original signal **E**. Each component of the row relates to one arc of the DMD. The arc of DMD reflects light into the photodiode when it corresponds to '1's, otherwise no light will be received by the detector. Fig. 1 showed a case of the operation, where the white and black arcs represent the ones in condition '1' and '0', respectively. In this way, the arcs on DMD and the photodiode provide one sampling denoted as follows

$$y_i = \varphi_i\boldsymbol{E} \quad i = 1, 2, \ldots, M, \quad (8)$$

where y_i is one of the M times samples **y**, φ_i is the i-th row of the $M \times N$ ($M \le N$) sensing matrix Φ. Eq. (8) is actually a sum of light energy of the diffraction pattern reflected by the detection arcs corresponding to '1's.

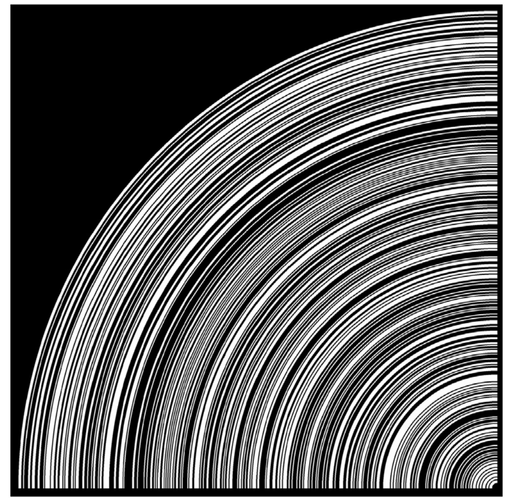

Fig. 1. The state of the detection arcs of DMD in one sampling.

From Eqs. (2) and (7), we have

$$\boldsymbol{y} = \Phi\boldsymbol{E} = [\Phi\boldsymbol{T}]\boldsymbol{W}, \quad (9)$$

By solving Eq. (9), the particle size distribution, i.e. **W**, can be retrieved by using the optimization method in CS theory.

3. Results and discussions

Three typical particle size distributions, i.e. Rosin-Rammler, lognormal and bimodal normal distributions, were used to compare the performance of detect arcs under different conditions in this paper. The Rosin-Rammler distribution is defined as

$$v(d) = \exp\left[-\left(\frac{d}{d_0}\right)^{N_s}\right]\frac{N_s}{d_0^{N_s}}d^{N_s-1}, \quad (10)$$

where d is the diameter of the particles, d_0 is central size of the distribution, and N_s is the spread of the distribution. Here, the default values are $d_0 = 46.3$ μm, $N_s = 4.1$. The lognormal distribution is defined as

$$v(d) = \frac{1}{d\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln d - \ln\overline{d}}{\sigma}\right)^2\right], \quad (11)$$

where $\overline{d}$ is the mean value of the particle diameters, and σ is the standard deviation. The default values are $\overline{d} = 46.3$ μm and $\sigma = 0.5$. The bimodal normal distribution is defined as

$$v(d) = \frac{1}{a\sigma_1\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{d-\overline{d_1}}{\sigma_1}\right)^2\right] + \frac{1}{b\sigma_2\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{d-\overline{d_2}}{\sigma_2}\right)^2\right], \quad (12)$$

where $\overline{d_1}$, $\overline{d_2}$, σ_1 and σ_2 are defined similar to the lognormal distribution. The default values are $a = b = 0.5$, $d_1 = 30$ μm, $d_2 = 70$ μm and $\sigma_1 = \sigma_2 = 10$.

The criterion for the performance of different configurations of the detect arcs is defined as the ratio of root mean square error of the retrieved distribution to the original distribution [16], termed as s-value,

Please cite this article as: J. Zhou, et al., Compressive sensing for particle size retrieval by using a digital micro-mirror device-based detector, Powder Technol. (2016), http://dx.doi.org/10.1016/j.powtec.2016.08.020