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Behavior of laminated shell composite with imperfect contact between the layers

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ABSTRACT

The paper focuses on the calculation of the effective elastic properties of a laminated composite shell with imperfect contact between the layers. To achieve this goal, first the two-scale asymptotic homogenization method (AHM) is applied to derive the solutions for the local problems and to obtain the effective elastic properties of a two-layer spherical shell with imperfect contact between the layers. The results are compared with the numerical solution obtained by finite elements method (FEM). The limit case of a laminate shell composite with perfect contact at the interface is recovered. Second, the elastic properties of a spherical heterogeneous structure with isotropic periodic microstructure and imperfect contact is analyzed with the spherical assemblage model (SAM). The homogenized equilibrium equation for a spherical composite is solved using AHM and the results are compared with the exact analytical solution obtained with SAM.

1. Introduction

Composite materials have emerged as the materials of choice in various branches of industry – aerospace, automotive, sport, etc. – for increasing the performance and reducing the weight and cost. However, defects induced during the manufacturing process or accumulated due to environmental and operational loads lead to the reduction in the mechanical performance and material strength and are recognized as a general problem in this type of composites, [1]. Most typically, such defects can be found at the interfaces between the layers creating an imperfect contact condition, [2,3].

The effect of the contact imperfectness on elastic properties of composites attracted the attention of researchers from 1970's, [4,5]. In [6–9], the authors obtained analytical expressions for the effective elastic properties of rectangular fibrous composites with imperfect contact between the matrix and the reinforcement. On the other hand, the multilayered curvilinear shell structures have received special attention in the last years. In [10–13] several mathematical methods have been used to derive analytical expression for the elastic properties of laminated shell composites. As a particular case, in [14], the expression of the effective coefficients for a curvilinear shell composite with perfect contact at the interface is obtained.

Several mathematical models and techniques have been developed to evaluate the elastic properties of curvilinear laminated shell composites with imperfect contact at the interfaces. In papers as [10,12,15–20], the assemblage model, finite elements method and the two-scale asymptotic homogenization method are used to derive in one way or another the effective behavior of the elastic properties of particular composites with imperfect contact at the interface.

In this paper a spherical shell structure is studied. In [21] the authors considered the effect in the elastic properties of a spherical laminated shell composite under the influence of stress and strain

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distributions for two composites with perfect and imperfect contact at the interface using AHM and SAM. In [21] the imperfect contact condition is modeled considering a thin interphase between the layers of the composite, i.e., a three phase composite is used in the analysis. Here the same effect in the spherical shell structure is studied except that the imperfect contact condition is modeled as a linear spring type and the FEM is used to validate the results obtained via AHM and SAM. The purpose of studying this kind of spherical structures obeys to the development and application of mathematical methods for the study of the cornea and other similar soft tissues.

In the present paper, first the AHM technique is used to evaluate the elastic properties of a two-layer laminated shell with imperfect contact of the spring type at the interface. The general analytical expressions of the effective coefficients are derived from the solution of the local problem. We focus on a two-layer spherical shell subjected to internal pressure assuming that the layers are isotropic. To validate the model, the effective coefficients of the spherical structure are compared with FEM calculations. The elastic fields (stresses, strains and displacements) are also compared with ones calculated by the method of Bufler [22] for the analysis of a spherical assemblage model (SAM). The approach is based on the transfer matrix method and yields closed form calculation of the equivalent elastic properties of a periodically laminated hollow sphere made of alternating layers of isotropic elastic materials with imperfect contact. The effective displacement, radial and hoop stresses computed via AHM are compared with the elastic fields calculated by FEM and SAM.

2. The linear elastic problem

A curvilinear elastic periodic composite is studied. The geometry of the structure is described by the curvilinear coordinates system $\mathbf{x} = (x_1, x_2, x_3) \in \Omega \subset \mathbb{R}^3$, where $\Omega = \Omega_1 \cup \Omega_2$ is the region occupied by the solid, it is bounded by the surface $\partial\Omega = \Sigma_1 \cup \Sigma_2$, where $\Sigma_1 \cap \Sigma_2 = \emptyset$, Ω_α $\alpha = 1, 2$ are the elements of the composite, separated by the interface Γ^ε. In Ω, the stress $\boldsymbol{\sigma}$ and strain $\boldsymbol{\epsilon}$ are related through the Hooke's law, $\sigma^{ij} = C^{ijkl}\epsilon_{kl}$, where C^{ijkl} are the components of the elastic tensor $\boldsymbol{C}$. For a linear periodic solid structure, the elastic tensor $\boldsymbol{C} \equiv \boldsymbol{C}(\mathbf{x}, \mathbf{y})$ is regular with respect to the slow variable $\mathbf{x}$ and $\mathbf{Y}$-periodic with respect to the fast variable $\mathbf{y} = \mathbf{x}/\varepsilon \in \mathbf{Y}$, where $0 < \varepsilon \ll 1$ characterizes the periodicity of the composite and $\mathbf{Y}$ denotes the periodic cell.

The linear elastic equilibrium equation for a curvilinear laminated shell composite with imperfect contact (spring type) at the interface is

$$\sigma^{ij}_{\;j} + \Gamma^i_{jk}\sigma^{kj} + \Gamma^j_{jk}\sigma^{ik} + f^i = 0, \quad \text{in} \quad \Omega, \tag{1}$$

subject to boundary conditions,

$$u_i = u_i^0 \quad \text{on} \quad \Sigma_1, \qquad \sigma^{ij}n_j = S^i \quad \text{on} \quad \Sigma_2, \tag{2}$$

and interface contact conditions,

$$\sigma^{ij}n_j = K^{ij}[[u_j]], \quad [[\sigma^{ij}n_j]] = 0, \quad \text{on} \quad \Gamma^\varepsilon, \tag{3}$$

where $\{\cdot\}_{,j} = \frac{\partial}{\partial x_j}\{\cdot\}$ is the derivative with respect to the slow curvilinear coordinate, Γ^i_{jk} are the Christoffel's symbols of second type, $[[\cdot]] = (\cdot)^{(2)} - (\cdot)^{(1)}$ denotes the jump at the interface Γ^ε, n_j is the normal vector to the corresponding surface $(\Sigma_2,\ \Gamma^\varepsilon)$, K^{ij} are the components of a matrix $\mathbf{K}$, that characterizes the imperfect contact in Γ^ε and the order of $\mathbf{K}$ is $O(\varepsilon^{-1})$. Replacing the Hooke's law and considering the Cauchy's formula, $\epsilon_{ij} = (u_{i,j} + u_{j,i})/2$, the equations (1)–(3) can be rewritten for the displacement vector function [14].

3. Homogenization of two-layer laminated shell composites with imperfect contact

In order to obtain an equivalent problem to (1)–(3) with not fast oscillating coefficients, the two–scales Asymptotic Homogenization Method (AHM) is used. The general expression of the truncated expansion is given by

$$u_m^{(\varepsilon)} = v_m + \varepsilon\left[\hat{N}^p_m v_p + N^{lk}_m v_{l,k}\right] + o(\varepsilon), \tag{4}$$

where $v_m \equiv v_m(\mathbf{x})$, $N^{lk}_m \equiv N^{lk}_m(\mathbf{x}, \mathbf{y})$ is the local function for the first order approach, $N^{lk}_{(1)m}(\mathbf{x}, \mathbf{y})$ is $\mathbf{Y}$-periodic, where $\mathbf{Y} = [0, 1]$ and $\hat{N}^p_m = -\Gamma^p_{lk}N^{lk}_m$ [14]. Substituting the expansion (4) into the Eqs. (1)–(3) a recurrent family of problem is obtained for different powers of the small parameter ε.

Considering a two-layer laminated shell composite with isotropic components, i.e.

$$C^{ijkl} = \lambda(y)g^{ij}g^{kl} + \mu(y)\left(g^{lj}g^{ki} + g^{il}g^{kj}\right), \tag{5}$$

where $[g^{ij}]$ is the metric tensor of the coordinates (x_1, x_2, x_3) and

$$\lambda(y) = \begin{cases} \lambda_1 & y \in [0, \gamma) \\ \lambda_2 & y \in (\gamma, 1] \end{cases}, \qquad \mu(y) = \begin{cases} \mu_1 & y \in [0, \gamma) \\ \mu_2 & y \in (\gamma, 1] \end{cases},$$

where the layers are transversal to the axis x_3, the local problem is obtained for ε^{-1}

$$\partial/\partial y\left(C^{i3lk} + C^{i3m3}\partial N^{lk}_m/\partial y\right) = 0 \quad \text{on} \quad \mathbf{Y} = [0, \gamma) \cup \{\gamma\} \cup (\gamma, 1], \tag{6}$$

with interface conditions given by the expressions

$$\left[C^{i3lk} + C^{i3m3}\partial N^{lk}_{(1)m}/\partial y\right] = (-1)^{\alpha+1}K^{ij}\left[\left[N^{lk}_{(1)j}\right]\right] \quad \text{on} \quad \Gamma^\varepsilon = \{y = \gamma\}, \tag{7}$$

$$\left[\left[C^{i3lk} + C^{i3m3}\partial N^{lk}_{(1)m}/\partial y\right]\right] = 0 \quad \text{on} \quad \Gamma^\varepsilon = \{y = \gamma\}, \tag{8}$$

where the parameter $\alpha = 1, 2$ denotes the layer.

Substituting (5) into the local problem (6) the following expression is obtained $\partial^2 N^{lk}_{(1)m}/\partial y^2 = 0$. Therefore, the local function has the expression

$$N^{lk}_m = \begin{cases} A^{lk(1)}_m y + B^{lk(1)}_m, & y \in [0, \gamma), \\ A^{lk(2)}_m y + B^{lk(2)}_m, & y \in (\gamma, 1]. \end{cases} \tag{9}$$

Considering the periodicity of the functions N^{lk}_m and $\partial N^{lk}_m/\partial y$ the following linear equations system is obtained from Eq. (8)

$$\left[C^{i3lk(1)} + C^{i3m3(1)}A^{lk(1)}_m\right] = -K_{im}\left(A^{lk(1)}_m(\gamma) + A^{lk(2)}_m(1-\gamma)\right), \tag{10}$$

$$\left[C^{i3lk(2)} + C^{i3m3(2)}A^{lk(2)}_m\right] = -K_{im}\left(A^{lk(1)}_m(\gamma) + A^{lk(2)}_m(1-\gamma)\right), \tag{11}$$

where the supraindex (α) $\alpha = 1, 2$ refers to each layer α. The linear problem (10) and (11) related to the variables $A^{lk(\alpha)}_m$ can be solved using classical methods and therefore the local functions are obtained.

Applying the average operator to the coefficient of the parameter ε^0, the homogenized coefficients are obtained and the general expression is given in the Eqs. (12)–(18) of [14]. The effective coefficients for a two-layer laminated shell composite with isotropic layers and imperfect contact condition at the interface have the general analytic expression

$$\widehat{h}^{ijkl} = \left\langle C^{ijkl}\right\rangle + V_1 C^{ijm3(1)}\frac{\partial N^{kl(1)}_m}{\partial y} + V_2 C^{ijm3(2)}\frac{\partial N^{kl(2)}_m}{\partial y}. \tag{12}$$

where V_α is the volume of the layers of the composite and the local functions $\partial N^{kl(\alpha)}_m/\partial y$ have the expression