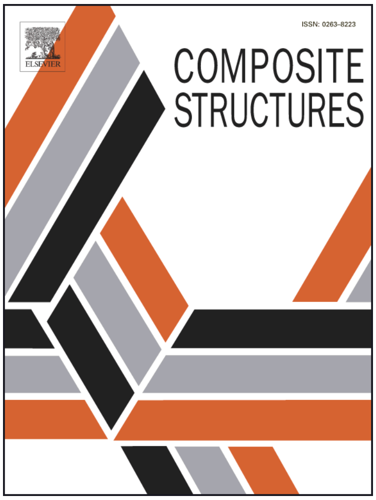

Bending stiffness of transversal isotropic materials

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Bending stiffness of transversal isotropic materials

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Abstract

Quasi-static bending of anisotropic and macroscopic homogeneous materials is studied as a two-dimensional elasticity problem. We extend a solution for beam-bending of isotropic materials found in the literature [1] , where our selected form of a transversal isotropic material allows for a scalar Airy stress function which demonstrates exactness of the interior solution. We also consider the plane-strain assumption to simulate cylindrical plate bending. However, violation of the essential boundary condition of the axial component of the inadmissible displacement field increases with increasing degree of anisotropy and we show that minimization of the violation in an average sense is effected by rigid-body rotation. Calculated center-line deflections reproduce Timoshenko's beam theory and its equivalent for cylindrical plate bending. The shear-correction factors of both depend on the degree of anisotropy and on Poisson's ratio. The accuracy of the theories is verified by comparison with FEM simulations. Finally, we address measurement of Young's modulus from standard three-point-bending tests and suggest that, given the typical specimen's rectangular cross-sectional aspect-ratios where the width is much larger than the height, an evaluation formula based on the plane-strain assumption gives more accurate results than the formulas suggested in the literature, which are based on the plane-stress assumption.

Keywords: two-dimensional elasticity problems, beam theory, anisotropic materials, three-point-bending tests

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1. Introduction

A recent review of structural behavior of laminated composite and sandwich beams by Sayyad and Ghugal [2] includes the development of theories for the bending of isotropic beams and of theories for the bending of laminated and sandwich beams. The development of theories for bending of beams starts with the Euler-Bernoulli classical beam theory (CBT) [3, 4]. It assumes that cross-sections remain plane and perpendicular to the deformed beam center-line and is therefore justified for describing the static response of beams whose length is much larger than their height. Higher eigenmodes of the dynamic response even of thin beams create locally thick-beam deformations where the influence of shear deformation becomes more pronounced which prompted Timoshenko to extend beam theory to include shear [5]. His first-order shear theory or Timoshenko beam theory (TBT) allows cross-sections to rotate with respect to the deformed beam center-line but holds on to the assumptions that they remain straight. As pointed out by Levinson [6], the resulting stress fields violate the traction-free natural boundary condition on the lateral surfaces of the beam. Shear-correction factors invented by Timoshenko [5, 7] and other authors [8, 9, 10, 11, 12, 13, 14] mitigate the problem that the first-order-shear theory per se cannot predict stiffness accurately. Two-dimensional elasticity theory identifies solutions satisfying the traction-free boundary conditions on the lateral beam surfaces. In a first step, one obtains an exact solution in terms of simple formulas for displacement and state-variables fields describing cross-sectional warping with a polynomial function of order three of the thickness coordinate z [1]. This solution is sometimes called incomplete because it can satisfy an essential boundary condition $u_x(z) = 0$ at some but not all points through the thickness. The so-called complete solutions can be seen as superposition of a closed-from exact solution and correcting solutions for removing the violation of the essential boundary condition. As the latter decays away from the location of the essential boundary condition, it is called an exterior solution in contrast to the incomplete, or interior domain, solution. The corrective solutions are in terms

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