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Identifying a rigidity function distributed in static composite beam by the boundary functional method

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ABSTRACT

In many situations, it is important to recognize the bending performance of a newly received composite beam. With this in mind, this paper estimates unknown rigidity function of a static Euler–Bernoulli composite beam. To solve the inverse coefficient problem with the help of boundary data, a sequence of boundary functions are derived, which satisfy the homogeneous boundary conditions, and are at least the fourth-order polynomials. All boundary functions and zero element constitute a linear space. An energetic boundary functional is introduced in the linear space, of which the energy is preserved for each energetic boundary function. The linear system used to recover the unknown rigidity function of composite beam with energetic boundary functions as bases is derived and the iterative algorithm is developed, which is convergent very fast. The accuracy and robustness of boundary functional method (BFM) are confirmed by comparing the estimated results with exact rigidity functions of composite beams.

1. Introduction

In structural engineering, aerospace vehicles and building components, beams are the most popular basic elements primarily to resist bending moment [1,2]. The forced vibration problems of beams have been many applications in building structure, mechanical cutting tool and aircraft engineering [3,4]. Sometimes one may encounter the problem that when the rigidity function of the beam is not yet known one wants to seek the solution of the beam, which results to an inverse coefficient problem of beam. The inverse coefficient problems of beams have been studied in many researches [5–9].

In recent decades, the composite beams with variable cross-section and interface have received much attention for they are needed to achieve a superior structural performance to fit the stress distribution better. Composite laminates are greatly being applied in many fields of engineering for their excellent properties of lightweight and fatigue resistance, and high strength and stiffness [10]. For the same reason that composite beams play as lightweight load carrying structures in varying applications, the most significant topic emerging from composite structural design is the improvement of rigidity performance in vibration analysis. Hajianmaleki and Qatu [11] have made a review focusing on the researches in last two decades on the vibration analysis of composite beams. To enhance the bending performance, there presented non-uniform constituent cross-section properties with interface between layered beams. One of the configurations of composite beam is the widely used scarf joint technique for fabricating composite structures [12,13]. Some related issues about the stability and behavior analysis of composite beams and laminated box beams can refer [14–21]. In all these situations, it is important to recognize the bending rigidity performance of a newly received composite beam by identifying its rigidity function [22].

For the inverse coefficient problem of static Euler–Bernoulli beam the measurement error of data may lead to a large discrepancy from the true rigidity distribution. In the present paper, the new method is based on a new idea of boundary functions for each beam under different boundary conditions. The boundary functions method is first developed by Liu and Li [9] to find the natural frequencies of non-uniform composite beam, where the Rayleigh quotient is maximized in terms of boundary functions. The present identification technique would be much data saving and time saving in the solution of the inverse coefficient problem of composite beam.

The remainder of this paper is arranged as follows. A new concept of energy functional in terms of boundary functions is introduced in Section 2, which constitute a linear space of all

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polynomial functions with at least fourth-order, and satisfy the homogeneous boundary conditions. Section 3 derives the iterative algorithm to recover the unknown rigidity function and several examples of simply supported beam are given in Section 4. Extending the new method and iterative algorithm for simply supported beam, an example of clamped-hinged beam is given in Section 5. A piecewise formulation of the boundary functional method is derived in Section 6 for identifying more complex rigidity functions of composite beams. Finally, the conclusions are drawn in Section 7.

2. Energetic functional of boundary functions

An inverse coefficient problem to find the unknown rigidity function $R(x)$ of a static Euler–Bernoulli composite beam is considered in this paper. The static beam with simply supported boundary conditions is employed as the first demonstrative example, of which one needs to find the static displacement distribution $y(x)$ as well as the rigidity function $R(x)$ that simultaneously satisfy

$$[R(x)y''(x)]'' = F(x), \quad 0 < x < \ell, \tag{1}$$

$$y(0) = 0, \quad y(\ell) = 0, \quad y''(0) = 0, \quad y''(\ell) = 0, \tag{2}$$

where the prime denotes the differential with respect to x, $F(x)$ is a given external loading, $R(x) > 0$ is an unknown rigidity function to be recovered, and ℓ is the length of beam.

Because the above problem has an unknown rigidity function $R(x)$, it cannot be solved directly to find $y(x)$. In order to recover $R(x)$, the over-specified data are given by

$$R(0) = R_0, \ R(\ell) = R_\ell, \quad R'(0) = R'_0, \quad R'(\ell) = R'_\ell, \tag{3}$$

which may be polluted by noise with

$$\begin{aligned} \overline{R}_0 &= R_0 + sr, \quad \overline{R}_\ell = R_\ell + sr, \\ \overline{R}'_0 &= R'_0 + sr, \quad \overline{R}'_\ell = R'_\ell + sr, \end{aligned} \tag{4}$$

where r is a random number and s is the intensity of noise.

By using Eqs. (1)–(3) to recover the unknown rigidity function $R(x)$ is a very difficult task, because the system (1)–(3) is seriously under-determined, and the resulting inverse coefficient problem is severely ill-posed. The new method will base on the new idea of boundary functions for each beam under different boundary conditions. Before that some basic mathematical ingredients are introduced in this section.

By multiplying both the sides of Eq. (1) by $y(x)$, yields

$$[R(x)y''(x)]''y(x) - F(x)y(x) = 0. \tag{5}$$

Integrating it from $x = 0$ to $x = \ell$ and the first term through integration by parts twice one can derive

$$\int_0^\ell R(x)y''(x)^2 dx - \int_0^\ell F(x)y(x)dx = 0, \tag{6}$$

where the boundary conditions in Eq. (2) were used. If there exist exact solutions $y(x)$ and $R(x)$ of Eq. (5), they must satisfy the above equation. The resulting equation is an energy equation and the author will use this energy functional as a mathematical tool to identify $R(x)$.

Really, one cannot exactly know $y(x)$ in Eq. (6), because $R(x)$ is an unknown function to be determined. However, one can set up some functions to approximate $y(x)$. First, the boundary function, which automatically satisfies the boundary conditions in Eq. (2), can be derived [9]:

$$\begin{aligned} B_1(x) &= x^4 - 2\ell x^3 + \ell^3 x, \ j = 1, \\ B_j(x) &= x^{j+3} - \frac{2j+3}{j+1}\ell x^{j+2} + \frac{j+2}{j+1}\ell^2 x^{j+1}, \ j \geqslant 2. \end{aligned} \tag{7}$$

They are at least fourth-order polynomial functions, satisfying the following homogeneous boundary conditions for the simply supported beam:

$$B_j(0) = 0, \quad B_j(\ell) = 0, \quad B''_j(0) = 0, \quad B''_j(\ell) = 0, \ j \geqslant 1. \tag{8}$$

A polynomial function of x is said to be a boundary function if it satisfies the homogeneous boundary conditions in Eq. (8). From Eq. (8) it is obvious that when $B_j(x)$ is a boundary function, $\beta B_j(x)$, $\beta \in \mathbb{R}$ is also a boundary function, and when $B_j(x)$ and $B_k(x)$ are boundary functions, $B_j(x) + B_k(x)$ is also a boundary function. The boundary functions are closure under a scalar multiplication and addition. Therefore, the set of

$$\{B_j(x)\}, \quad j \geqslant 1 \tag{9}$$

and the zero element constitute a linear space of boundary functions, denoted by $\mathcal{B}$.

The following result is important to help us identify the unknown rigidity function $R(x)$ of composite beam.

Theorem 1. *In the linear space $\mathcal{B}$ there exist homogeneous linear elements*:

$$E_j(x) = \gamma_j B_j(x) + B_{j+1}(x), \quad j \geqslant 1, \ j \text{ not summed}, \tag{10}$$

such that $R(x)$ is a solution of the following functional equation in terms of $E_j(x)$:

$$\int_0^\ell R(x)E''_j(x)^2 dx - \int_0^\ell F(x)E_j(x)dx = 0, \tag{11}$$

where

$$\begin{aligned} a_2 &= \int_0^\ell R(x)B''_j(x)^2 dx, \\ a_1 &= \int_0^\ell [2R(x)B''_{j+1}(x)B''_j(x) - F(x)B_j(x)]dx, \\ a_0 &= \int_0^\ell [R(x)B''_{j+1}(x)^2 - F(x)B_{j+1}(x)]dx, \end{aligned} \tag{12}$$

$$\gamma_j = \frac{-a_1 - \sqrt{a_1^2 - 4a_0 a_2}}{2a_2}. \tag{13}$$

Proof. Because $B_j(x), B_{j+1}(x) \in \mathcal{B}$ are elements of the linear space $\mathcal{B}$, the linear combination in Eq. (10) renders $E_j(x) \in \mathcal{B}$ also an element of the linear space $\mathcal{B}$, which satisfies the homogeneous boundary conditions for the simply supported beam:

$$E_j(0) = 0, \quad E_j(\ell) = 0, \quad E''_j(0) = 0, \quad E''_j(\ell) = 0, \tag{14}$$

due to Eq. (8).

Because $E_j(x)$ already satisfies the boundary conditions in Eq. (14) as Eq. (2) for $y(x)$, by turning the attention to the energy identity (6) one can approximate $y(x)$ by $E_j(x)$ and derive Eq. (11), which is an energetic boundary functional of $E_j(x)$ defined in the linear space.

Inserting Eq. (10) for $E_j(x)$ and

$$E''_j(x) = \gamma_j B''_j(x) + B''_{j+1}(x) \tag{15}$$

for $E''_j(x)$ into Eq. (11) one can derive a quadratic equation to determine the multiplier γ_j:

$$a_2\gamma_j^2 + a_1\gamma_j + a_0 = 0, \tag{16}$$

where the coefficients a_0, a_1, a_2 were defined in Eq. (12). Then the solution of γ_j is derived in Eq. (13). This ends the proof of this theorem. □