



A layer-wise theory of shallow shells with thin soft core for laminated glass and photovoltaic applications



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ABSTRACT

The aim of this paper is to develop a robust layer-wise theory for structural analysis of curved glass and photovoltaic panels. By the analogy to the existing theories of plates, governing equations for doubly curved layers including kinematic relations, equilibrium conditions and constitutive equations are introduced. Applying assumptions of shear rigidity of skin layers and moments-free core layer as well as approximations of thin shallow shell, a reduced form of governing differential equations is proposed. Compared to the classical theories of shells the derived system includes an additional second order differential equation. As a result, additional boundary conditions should be satisfied for any edge of the shell. The importance of these extensions is demonstrated for long cylindrical panel with for two examples of simple supports: one with free edges, where relative in-plane displacements of skins are allowed, and one with framed edges, where cross-section rotations of all layers are assumed the same. For both cases closed-form analytical solutions related to a shell strip approximation are presented. Displacement bounds in monolithic and layered cases are derived, and the dependence of deformation and stress characteristics on the radius of curvature and types of supports are illustrated.

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1. Introduction

Laminated shells with stiff skin layers and a relatively thin and soft core layer are integrated in many engineering structures. For example, laminated glass shells with glass skin layers and a core layer from Polyvinyl Butyral (PVB) and are widely used in civil engineering and automotive industry [1–5]. Crystalline or thin film photovoltaic modules are composed from front glass layer, solar cell layer embedded in a polymeric encapsulant and back glass or polymeric layer [6–8]. Lightweight photovoltaic modules replace front and/or back glasses by polymer or polymer composite layers [9]. In the last years curved photovoltaic modules are designed for the integrity in shell-type structures. Fig. 1 shows two examples from automotive industry and civil engineering.

One feature of laminated glass shells or laminates used in photovoltaic industry is a relatively high contrast of material properties for core and skin layers. As an example consider the ratio $\xi = \mu_c / \mu_s$ with μ_c being the shear modulus of the core layer and μ_s being the shear modulus of the skin layer. For laminated glass

and photovoltaic structures ξ ranges between 10^{-5} and 10^{-2} [6,10,11]. For comparison, in classical sandwich structures, ξ is in the range of 10^{-2} and 10^{-1} . Furthermore, in glass and photovoltaic laminates the face layers are relatively thick and the core layer is thin, while classical sandwiches are composed from thin skins and a relatively thick core.

Laminated glass panels and solar modules are subjected to mechanical and thermal loadings, such as snow, wind, impact loads and daily or seasonal temperature changes. For design it is beneficial to analyze the load transfer between the layers, to establish strain and stress states in plies and to predict deformation and strength of the laminate under given external loadings.

A robust approach for the structural analysis is the first order shear deformation theory (FSDT) of shells and plates [12–15]. Here the assumption is made that any normal to the reference mid-surface of the shell behaves like a rigid line during the deformation. The rotation degrees of freedom (DOFs) of the lines are independent from the displacements of the mid-surface, in contrast to the Kirchhoff–Love theory, where the lines are assumed normal to the deformed mid-surface and rotations are related to displacements. Closed-form solutions or approximate analytical solutions for beams plates and shells according to the FSDT are presented

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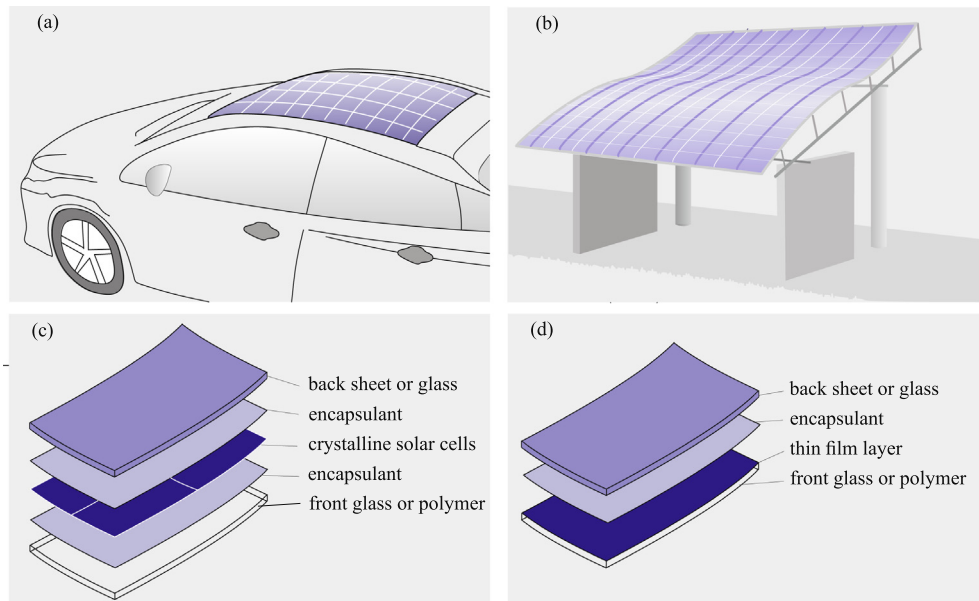


Fig. 1. Examples of curved photovoltaic modules. (a) Photovoltaic module on the roof of a car, (b) Photovoltaic integrated roof shading system, (c) Layered structure of crystalline photovoltaic module, (d) Layered structure of thin film photovoltaic module.

in [12,15–20], among others. Furthermore shell elements developed with FSDT are available in commercial finite element codes for the analysis of layered structures [21]. The key step in applying FSDT is to establish material properties of the laminate, in particular the transverse shear stiffness. Although closed-form relationships are developed to find effective elastic stiffness of a laminate from the properties of layers, e.g. [22], numerical techniques are required to estimate the effective transverse shear deformation in the inelastic range, [23–25]. Furthermore, for laminates with extreme differences in the stiffness of layers the FSDT fails to predict the deformation properties of the laminate accurately, as shown in [6,9] for beams and in [26–29] for plates.

Laminated glass shells can also be analyzed by the use of three-dimensional theory of elasticity and applying the solid finite elements for the numerical solution. However, due to extreme differences in material properties of plies and the relatively low thickness of the core, considerable numerical effort is required to obtain the results with a desired accuracy [6,26].

Continuum shell elements can also be applied for the analysis of laminated glass units and photovoltaic modules. The finite element code Abaqus, for example, offers continuum shell elements, which possess displacement DOFs and use three-dimensional constitutive equations [21]. They include the linear triangular and quadrilateral elements with 18 and 24 DOFs, respectively. However, at least three elements in the thickness direction are required to analyze a three-layer laminate. Therefore the total number of DOFs required for analysis of an entire laminate can increase considerably.

Recently higher order shell theories were developed and applied to the analysis of laminated structures. A widely used approach is the zig-zag theory, where the displacements are approximated by piecewise functions with respect to the thickness coordinate such that the compatibility between the layers is fulfilled. Applying these approximations the governing equations of the three-dimensional elasticity theory are reduced to two-dimensional shell equations by the use of variational methods or asymptotic techniques [30–35].

Within the layer-wise theory (LWT), balance equations and constitutive models are derived for each ply independently. With constitutive assumptions for interaction forces and compatibility

conditions between the layers a model for the laminate can be developed. One advantage of LWT is that the load transfer between the layers can be analyzed explicitly since forces of interactions are directly accessible, while within zig-zag-type approximations Lagrange multipliers appear to be reactions to the constraints. Furthermore, within LWT additional assumptions can be made with regard to kinematics and/or dynamics of individual layers leading to robust governing equations. For example, for curved laminated beams with core layer from soft polymers the assumption is made that glass skin layers deform according to the Bernoulli–Euler beam theory, i.e. they are shear-rigid. The soft core layer carries out the transverse shear stress only, while the bending moment and the normal force are negligible [1,3]. Recently, LWTs have been developed for laminated plates with soft and thin core layer [36,37]. The deformation of skin layers is described by the Kirchhoff plate theory, while the core is modeled as the shear layer. In [26,37] closed-form analytical solutions are presented for rectangular plates with various boundary conditions. In [10,26] a finite element is developed and utilized inside Abaqus for the analysis of laminates based on LWT. The element possesses nine DOFs: two mean in-plane displacements, two relative in-plane displacements, the deflection, two mean cross-section rotations, and two relative cross-section rotations.

More advanced theories may be required for shells with complex internal structure such as pantographic lattices or fiber sheets [38–40]. In addition multi-scale approaches are useful for photovoltaic laminates, where the global structural analysis is performed by means of LWT, while detailed analysis of constituents, like solar cells and interconnectors, is performed within a representative volume by the use of three-dimensional theory [8,10].

Despite the fact that curved laminate panels can be analyzed by many different approaches, equations of LWT provide a mostly efficient tool in a design stage of laminated glass and photovoltaic structures, since closed form analytical solutions for stress and deformation states can be derived and influence of many parameters like layer thickness, material properties and curvature can be analyzed explicitly. Although LWTs for curved beams and plates are widely discussed in the literature, robust equations for laminated shells with soft core layer are not available, to best of our knowledge.

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