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Three-dimensional modeling of the wave dynamics of tensegrity lattices

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ABSTRACT

This paper develops effective numerical models to study the wave dynamics of highly nonlinear tensegrity metamaterials. Recent studies have highlighted the geometrically nonlinear response of structural lattices based on tensegrity prisms, which may gradually change their elastic response from stiffening to softening through the modification of mechanical, geometrical, and prestress variables. We here study the nonlinear dynamics of columns of tensegrity prisms subject to impulsive compressive loading. An effective nonlinear rigid body dynamics is employed to simulate the dynamic response of such metamaterials. We illustrate how to pass from the matrix to the vector form of the equations of motions, on accounting for a rigid response of the compressive members (bars). Numerical simulations show that the wave dynamics of the examined metamaterials supports compression solitary pulses with profile dependent on the elastic properties of the tensile members (strings), the given impact velocity, and the applied prestress. We conclude that tensegrity columns can be effectively used as tunable acoustic lenses, which are able to generate acoustic solitary waves with adjustable profile in a host medium.

1. Introduction

The dynamics of strongly nonlinear metamaterials is receiving increasing attention by the scientific community, (refer, e.g. to review papers [1,2] and references therein). Several studies have shown that elastically hardening discrete systems support compressive solitary waves and the unusual reflection of waves on material interfaces [3–8], while elastically softening systems support the propagation of rarefaction solitary waves under initially compressive impact loading [9,6]. Solitary wave dynamics has been proven to be useful for the construction of a variety of novel acoustic devices. These include: acoustic band gap materials; shock protector devices; acoustic lenses; and energy trapping containers, to name some examples (refer, e.g., to [10] and references therein).

It has been found that structural lattices based on tensegrity units (e.g., tensegrity prisms) exhibit a tunable geometrically nonlinear response, which may gradually change from stiffening to softening through the modification of mechanical, geometrical, and prestress variables [11–13,9]. Tensegrity structures are prestressable truss structures, obtained by connecting compressive members (bars or struts) through the use of pre-stretched tensile elements (cables or strings). It is known that tensegrity concepts diffusely appear in nature, such as, e.g., in cells, the structure of the spider silk, and the system of bones and tendons in animals and humans [14]. Attention is increasingly being given to the development of efficient analytical and numerical methods for exploiting tensegrity concepts in engineering design (refer to [14] and references therein). Also the form-finding of tensegrity structures continues to be an active research area, due to both their easy control (geometry, size, topology and prestress control) [15], and the fact that such structures provide minimum mass systems under different loading conditions [16–18,14,19,20]. The use of fractal geometries for the multiscale design of tensegrity systems - diffusely illustrated [20,21] – is of particular interest.

The importance of protecting materials and buildings against impacts with external objects is well known (cf., e.g., [4,22]). Equally, there is growing interest in research into noninvasive tools to target defects in materials, and for monitoring structural health in materials and structures [23–26]. In the past, shock protectors and devices used for focusing acoustic waves mainly relied on energy dissipation and the modification of sound propagation through spatially dependent delays. Highly efficient and unconventional mechanisms for protecting materials and focusing mechanical waves through the use of rarefaction and compression solitary waves have recently been discovered by [8,9]. It is worth noting that arrays of tensegrity lattices with elastically hardening response can be employed to fabricate tunable focus acoustic lenses that support extremely compact solitary waves [8,9].

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Three-dimensional finite element (FE) models of lattice structures usually make use of tetrahedral elements with a large number of degrees of freedom [27]. Such models are hardly applicable to dynamic simulations, even for lattices constituted by a small number of cells. A key goal of the present work is to develop efficient and accurate models of tensegrity lattices that make use of 3D assemblies of one-dimensional models for bars and strings. By describing the bars as rigid members and the cables as elastically deformable elements, we first develop the dynamics of an arbitrary tensegrity network in matrix form, and next we show how to switch such a formulation to vector form (Section 2). The latter proves to be useful in order to coupling the proposed model with standard FE models that may interact with tensegrity networks. The time-integration of the equations of motion is conducted through a Runge–Kutta algorithm that accounts for a rigidity constraint of the bars [28].

In Section 3, we apply the proposed numerical model to investigate the nonlinear wave dynamics of tensegrity columns. We study the dynamic response of such systems to impact loading, by establishing comparisons with the alternative model proposed in Ref. [29], which assumes rigid response of the terminal bases of each prism [30]. We show that our 3D modeling of tensegrity columns allows us to detect different strain wave profiles, as a function of the applied prestress, and a rigidity parameter describing the kinematics of the terminal bases (Section 4). Such tunable response can be profitably used to build adaptive arrays of tensegrity columns, which can subjected to different levels of prestress, so as to generate solitary waves with different phases that coalesce at a focal point in an adjacent host medium [23,24]. We draw the main conclusion of the present study and future research lines in Section 5.

2. A numerical model for the dynamics of tensegrity networks

The dynamic problem of a general three-dimensional lattice is hereafter presented via a suitable, vector form reformulation of the tensegrity dynamics presented in [31]. We first introduce some basic notation (Section 2.1). Next we summarize the matrix-form of the tensegrity dynamics diffusely illustrated in [31] (Section 2.2), and then pass to develop the vector form of the equations of motions employed in the present work (Section 2.3).

2.1. Basic notation

2.1.1. Matrices and vectors

Throughout the paper, we indicate matrices with bold capital letters (ie $\mathbf{X}$), vectors with bold lower case letters (i.e. $\mathbf{x}$), and scalars with italic letters (ie x).

For later use, we introduce the following operators:

- $\hat{\mathbf{x}} = diag(x_1,\ x_2,\ \ldots,\ x_n)$is an operator that produces a diagonal matrix with the components $x_1, x_2, \ldots, x_n$ of the vector $\mathbf{x}$;
- $\lfloor \mathbf{X} \rfloor$ is an operator that keeps only the diagonal terms of the square matrix $\mathbf{X}$ and set to zero all the off-diagonal terms;
- vec($\mathbf{X}$) indicates the vectorizing operator that stack up all columns of matrix $\mathbf{X}$;
- the Kronecker product between two matrices $\mathbf{A} \in \mathbb{R}^{m\times n}$ and $\mathbf{B} \in \mathbb{R}^{p\times q}$, through the equation:

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & \cdots & a_{1n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m1}\mathbf{B} & \cdots & a_{mn}\mathbf{B} \end{bmatrix} \in \mathbb{R}^{mp\times nq} \tag{1}$$

where a_{ij} is the ith,jth element of the matrix $\mathbf{A}$.

2.1.2. Tensegrity networks

Let us consider a tensegrity network made up of n_n nodes (or joints), n_b bars and n_s cables (Fig. 1). The joints are frictionless hinges, and each member carries only axial forces. The bars (i.e., the compressed members) are assumed to behave as straight rigid bodies (rods) with uniform mass density, constant cross-section, and negligible rotational inertia about the longitudinal axis. The cables are instead modeled as straight elastic springs that can carry only tensile forces.

The generic node i, with $i \in [1, \ldots, n_n]$, is located by the vector $\mathbf{n}_i \in \mathbb{R}^3$ in the three-dimensional Euclidean space, and is loaded with an external force vector $\mathbf{w}_i \in \mathbb{R}^3$. By suitably collecting the vectors $\mathbf{n}_i$ and $\mathbf{w}_i$, we introduce the following nodal and force matrices:

$$\mathbf{N} = [\,\mathbf{n}_1 \quad \mathbf{n}_2 \quad \ldots \quad \mathbf{n}_i \quad \ldots \quad \mathbf{n}_{n_n}\,] \in \mathbb{R}^{3\times n_n} \tag{2}$$

$$\mathbf{W} = [\,\mathbf{w}_1 \quad \mathbf{w}_2 \quad \ldots \quad \mathbf{w}_i \quad \ldots \quad \mathbf{w}_{n_n}\,] \in \mathbb{R}^{3\times n_n} \tag{3}$$

The kth bar (or cable) k of the network, with $k \in [1, \ldots, n_b]$ (or $k \in [1, \ldots, n_s]$), is located by the vector $\mathbf{b}_k \in \mathbb{R}^3$ (or $\mathbf{s}_k \in \mathbb{R}^3$). For example, if the kth bar connects nodes i and j, then $\mathbf{b}_k = \mathbf{n}_j - \mathbf{n}_i$. By stacking up the bar and string vectors, we obtain the following matrices describing the geometry of all bars and cables:

$$\mathbf{B} = [\,\mathbf{b}_1 \quad \mathbf{b}_2 \quad \ldots \quad \mathbf{b}_k \quad \ldots \quad \mathbf{b}_{n_b}\,] \in \mathbb{R}^{3\times n_b}, \tag{4}$$

$$\mathbf{S} = [\,\mathbf{s}_1 \quad \mathbf{s}_2 \quad \ldots \quad \mathbf{s}_k \quad \ldots \quad \mathbf{s}_{n_s}\,] \in \mathbb{R}^{3\times n_s} \tag{5}$$

The center of mass of the kth bar between nodes i and j is located by the vector $\mathbf{r}_k = (\mathbf{n}_i + \mathbf{n}_j)/2$. Collecting all the $\mathbf{r}_k$ vectors, we get the matrix:

$$\mathbf{R} = [\,\mathbf{r}_1 \quad \mathbf{r}_2 \quad \ldots \quad \mathbf{r}_k \quad \ldots \quad \mathbf{r}_{n_b}\,] \in \mathbb{R}^{3\times n_b} \tag{6}$$

It is useful to rewrite the above matrices as follows:

$$\mathbf{B} = \mathbf{N}\mathbf{C}_{\mathbf{B}}^{\mathrm{T}}, \qquad \mathbf{S} = \mathbf{N}\mathbf{C}_{\mathbf{S}}^{\mathrm{T}}, \qquad \mathbf{R} = \mathbf{N}\mathbf{C}_{\mathbf{R}}^{\mathrm{T}} \tag{7}$$

where $\mathbf{C}_{\mathbf{B}} \in \mathbb{R}^{n_b\times n_n}$ and $\mathbf{C}_{\mathbf{S}} \in \mathbb{R}^{n_s\times n_n}$ are *connectivity* matrices of bars and cables, respectively. The general ith row of $\mathbf{C}_{\mathbf{B}}$ (or $\mathbf{C}_{\mathbf{S}}$) corresponds to the ith bar (or cable), and the element $\mathbf{C}_{\mathbf{B}ij}$ (or $\mathbf{C}_{\mathbf{S}ij}$) is equal to: -1 if vector $\mathbf{b}_i$ (or $\mathbf{s}_i$) is directed away from node jth,1 if vector $\mathbf{b}_i$ (or $\mathbf{s}_i$) is directed toward node jth, and 0 if vector $\mathbf{b}_i$ (or $\mathbf{s}_i$) does not touch node j. Similarly, the ith row of $\mathbf{C}_{\mathbf{R}} \in \mathbb{R}^{n_b\times n_n}$ corresponds to the bar $\mathbf{b}_i$, and the element $\mathbf{C}_{\mathbf{R}ij}$ is equal to: 1 if vector $\mathbf{b}_i$ is touching node j, or 0 if vector $\mathbf{b}_i$ does not touch node j. Following Ref. [14], we say that a tensegrity network is of *class n*, if the maximum number of bars concurring in each node is equal to n.

2.1.3. Cable forces

Let us consider now the generic cable (say the kth one) with Young modulus of the material E_{sk}, cross-section area A_{sk}, rest length L_k, and stretched length s_k (i.e. $s_k = \|\mathbf{s}_k\|$, and $s_k \geqslant L_k$). We define the stiffness k_{sk} and the prestrain p_k through the following equations:

$$k_{sk} = \frac{E_{sk}A_{sk}}{L_k}, \tag{8}$$

$$p_k = \frac{s_k - L_k}{L_k} \tag{9}$$

The force density carried by the current cable is given by the following (unilateral) constitutive equation (*elastic, no-compression response*):

$$\gamma_k = \max\left[k_{sk}\left(1 - \frac{L_k}{s_k}\right), 0\right], \quad if : s_k \geqslant L_k, \tag{10}$$

$$\gamma_k = 0, \quad if : s_k < L_k \tag{11}$$