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Influence of the coupling matrix B on the interactive buckling of FML-FGM columns with closed cross-sections under axial compression

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To the Memory of Professor Katarzyna Kowal-Michalska – Professor Kasia

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ABSTRACT

The study of the coupled instabilities of thin-walled columns with trapezoidal and square cross-sections, which are made of Functionally Graded Materials (*FGMs*) and Fibre Metal Laminates (*FMLs*) is presented in the paper. It is assumed that each column wall is made of a stack of nine FML layers and/or a single FGM (Al-TiC) and/or ceramics (TiC). The GLARE 3 type FML part was made of an alternate sequence of Al 2024-T3 and even GFR prepreg layers where aluminium was always the outer layer. In the case of the FGM layer, volume fractions of ceramics and the metal distribution throughout the layer thickness are described by a simple power law. It is assumed that the columns are subjected to axial compression and simple supported at their loaded edges. All constituent materials obey Hooke's law. The effect of temperature influence is neglected. To determined governing equations of considered FML-FGM structures the classical laminate plate theory (CLPT) is used. The solution to the problem of the non-linear buckling of hybrid thin-walled structure is based on the Koiter's theory. An interaction of the global buckling mode with two local buckling modes is taken into account. In order to derive the equilibrium equations of FML-FGM profiles, the full Green's strain tensor and the second Piola-Kirchhoff's stress tensor have been adapted. The presented solution seems to be especially important as the authors have not found no earlier studies on the coupled buckling of thin-walled FML-FGM structures with closed cross-sections.

1. Introduction

This paper is an effect of the late Professor Kowal-Michalska's idea to investigate thin-walled structures combined of FGM and FML-type materials [15]. Inspired by this field of interest, we are dedicating this article to Her memory. This paper is also an extension of [20] on the combination of FML and FGM materials into the thin-walled structure engineering design.

In the recent thirty years Functionally Graded Materials (*FGMs*) and Fibre Metal Laminates (*FMLs*) have been a relatively new class of composite materials, which have become a very popular research field and have been used in numerous engineering applications. The buckling and post-buckling behaviour of conservative systems have been present for a number of years within the scientific investigations. The local buckling causes a harmful reduction in stiffness of the section and consecutive lowering of load carrying capacity with respect to the non-locally buckled section whereas the global buckling mode leads always to collapse of thin-walled structures. The problem of an interaction of global modes and the local ones is of great significance in thin-walled structure analysis.

In the literature on interactive buckling, there are hardly any works devoted to functionally graded structures (the so-called FG structures) except for [20,21]. The investigation of interactive buckling (i.e., coupled buckling) requires the general non-linear theory of stability application, in which Koiter's theory [11] is the most popular one. The Byskov and Hutchinson theory [5] is based on an asymptotic Koiter's type expansion of the post-buckling path and is capable of considering a few buckling modes. The determination of the post-buckling equilibrium path requires the second order approximation to be taken into account (for a more detailed analysis, see [13,18,20]).

FGMs are inhomogeneous new class composites made up of two constituents phases: metallic and ceramic one. The volume fracture of both phases changes gradually along the thickness of the structures giving a smooth, gradual change of material properties. In [4], Birman and Byrd give a wide review of theories which are mostly employed to describe grading material properties. The comparison of numerical results of the linear third-order theory (TSDT), non-linear first-order theory (FSDT) and classical laminated plate theory (CLPT) applied to Navier solution of rectangular

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FGM plates gives Reddy in [28]. The static and dynamic analyses show that the discrepancy between FSDT and CLPT is of 2% in the plate deflections and even indistinguishable for some other results depending on components distribution.

In [19] Kolakowski et al. show – on the basis of Koiter's non-linear theory, that FG plates have non-symmetric stable post-buckling equilibrium paths. Both – the semi-analytical method (SAM) and finite element method (FEM) numerical results compared – have proved this statement. This feature explains the differences in a plate response dependence on the imperfection sign (sense). A FG plate has a non-trivial coupling matrix B [19]. That means the plate out-of-flatness appears from the very beginning of compressive loading and the bending moment at loaded edges is not equal to zero [2]. Therefore, in several papers (e.g., [37]), a concept of the 'physical neutral surface' is introduced which allows one to uncouple the in-plane and out-of-plane deformations. Then the solution becomes as easy as for the homogenous isotropic plate.

The development of theories and methods applied to the interactive buckling analysis of thin-walled structures can be traced in many papers, e.g., [12,16,17,22,31]. However, the greatest number of these publications describe a buckling and post-buckling behaviour of columns made of isotropic materials. One can find only a few papers in which the buckling and post-buckling response of thin-walled elements made of FGMs under compression (for plates [9,30]) is presented. The non-linear analysis of plates and shells devoted to basic types of loads is covered in the monograph by Hui-Shen [8].

Recently Kolakowski and Teter studied in [20], the coupling buckling of thin-walled columns of closed – trapezoidal and square, cross-sections which walls were made of functionally graded material. A special attention has been focused at the influence of the imperfection sign (sense) and the character of post-buckling equilibrium path on the load carrying capacity. The different behaviour of considered columns in the post-buckling range can be easily explained by a direct effect of their cross-section shapes and by the non-symmetric stable post-buckling path for FG structures.

In the case of FG plates under compound mechanical and/or thermal loads due to their complexity, in a great number of buckling problems the FEM gives particular possibility for solution and seems to be the only.

The next group of advanced composite materials, mentioned at the beginning, are Fibre Metal Laminates (FMLs) [1]. FMLs are hybrid materials, built of thin layers of metal alloy and fibre reinforced epoxy resin. These materials are manufactured by bonding composite plies to the metal ones [36]. FMLs, as regards metal layers, can be divided into FMLs based on aluminium alloys (ARALL reinforced with aramid fibres, GLARE – glass fibres, CARALL – carbon fibres) and others. Nowadays, materials such as GLARE (glass fibre/aluminium), due to their very good fatigue and strength characteristics combined with their low density, find increasing applications in aircraft industry [34].

GLARE laminate is built of alternate aluminium sheets and unidirectional high-strength glass fibre layers pre-impregnated with adhesive. Each glass-reinforced plastic layer can be made up of a certain number of unidirectional plies, which are stacked either unidirectionally or in a cross-ply or angle-ply arrangement. The number, orientations, and the stacking sequence of unidirectional plies in the composite layer depend on the GLARE commercial grade. Thus the number of glass-reinforced layers may differ but the outer layers are always aluminium and the number of glass fibre layers is always one less. The most common type of aluminium applied in GLARE is the 2024-T3 Alloy. The overall laminate is mostly symmetric. In current production parts such as the fuselage of the largest civil transport aircraft, it has been introduced in primary structure applications. GLARE™ material properties, as well as FMLs in general, exhibit partly metallic and partly composite behaviour. The hybrid nature of FMLs has an advantage of lower density when compared with monolithic aluminium fuselage skins, but which is even more important, it exhibits a natural crack arresting capability due to fibre layers in the presence of a fatigue crack, which is a major concern in the design of monolithic aluminium. The hybridization of materials in multi-layered structures leads inevitably to a decrease in the buckling load capacity, which is only partly off-set by a weight reduction.

The objective of the current study is also to reveal improvements in the compression buckling strength of the analysed thin-walled profiles through the use of different hybrid FML strategies. Due to improvement of fatigue resistance a lower thickness and/or higher stresses in GLARE structural elements have been accepted. Therefore an instability phenomena become one of the most important design constraints and need to be satisfied for safety reasons [35]. This issue has always been one of the main field of scientists' interest which especially concerns thin-walled structures. Various analyses have been performed also to study the static buckling of FMLs [3,26]. It should be emphasized that the list of publications devoted to the buckling and post-buckling response of thin-walled FML panels is limited. In many papers buckling tests of composite thin-walled members performed in the laboratory are compared to the FEA and an optimizing analysis might be conducted on this basis. For example this approach of stability investigations is present in a few papers [6,7,16]. There a semi-analytical method is employed to validate the results in the buckling and post-buckling state. Numerous analyses of thin-walled columns have also compared the buckling behaviour of a variety of complex open cross-section profiles/columns and different fibre alignments [1]. As a preliminary analysis to the FML buckling study can be taken [24], where various column shapes, a selection of material constituents as well as experimental procedures are discussed. It is well known that real structures posses usually even inconsiderable imperfections in comparison to the ideal ones, hence it has been proven that the results obtained from the experiment differ to some extent from the theory and the analytical solution [25].

In this study, both the global and local buckling modes in the response of FML-FGM plate structures subjected to axial compression have been taken into account. A plate model is adopted for the entire FML-FGM panels and the governing equations of the thin-walled FML-FGM plate equilibrium are determined based on the classical laminate plate theory (CLPT). The thermal effects are neglected. To obtain the differential equilibrium equations of individual plates with a variational method for the asymptotic analytical-numerical method, the non-linear theory of composite plates has been modified in such a way that the full Green's strain tensor for thin-walled plates, the second Piola-Kirchhoff's stress tensor in Lagrange's description and a numerical method of the transition matrix using Godunov's orthogonalization have been taken into account [14,16,17]. The applied procedure allows the analysis of all buckling mode interactions. Employing this methodology to FGM plates and thin-walled columns Kolakowski et al. [19,20] proved the non-symmetric post-buckling equilibrium path of considered structures. Thus the present work deals with an influence of the non-trivial coupling matrix B on the interactive buckling of thin-walled FML-FGM columns with trapezoidal and square cross-sections. In the very recent literature one can find first few papers devoted to buckling of thin-walled compound structures made of both FML and FGM layers.

2. Analytical background

Under consideration there are long thin-walled prismatic columns of the length L, composed of plane, rectangular plate