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Strain distributions in superconducting strands with twisted filaments

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ABSTRACT

The aim of this article is to analyze the strain distributions of filament bundles in the superconducting strands subjected to external mechanical loads and thermal strain. A multistage micromechanical model is adopted to characterize the mechanical behavior of the superconducting strand with twisted filaments. First, we employ the equivalent model to simplify the filament bundles. Then, the continuous filament bundles are divided into discrete elements, and an incremental mean-field micromechanics theme based on the Mori-Tanaka method is used to obtain the equivalent properties of the strand and the local strains in the Nb_3Sn filament bundles. The results for two different structures are compared. The variation trends of normal strains along *x*, *y* and *z* directions are different. The effects of temperature on the equivalent properties of strands are discussed.

1. Introduction

Nb_3Sn is one of the low temperature superconducting materials which are used in International Thermonuclear Experimental Reactor (ITER) [1]. The peak magnetic field of 13T will be provided by the cables composed of Nb_3Sn strands in central solenoid coils [2]. The critical transition temperature of Nb_3Sn is 18 K and the magnetic field can up to 25T in the liquid helium temperature [3]. Therefore, Nb_3Sn strand can be used for a relatively high magnetic field environment [3,4]. However, Nb_3Sn is a typical brittle material which cannot be extruded and drawn directly. The preparation process is complex and the coil will be wound before the reaction heat treatment [5]. The reaction temperature of the entire coil is about 900–950 K [6]. Since the Nb_3Sn strand is made of bronze, copper and Nb3Sn filaments which have different thermal expansion coefficients, a larger thermal shrinkage will occur as the sample is cooled from the reaction temperature to the room temperature or operating temperature. Then the strand will experience the intrinsic thermal strain after heat treatment [7–9]. In addition, the strand is usually subjected to a high electromagnetic force. The filaments of strand may be damaged under larger strain or stress [10–12]. It is well known that the strain sensitivity of Nb3Sn strand may cause the degradation of performance [6]. Therefore, the prediction for the strain field of filaments is very important in the design of superconducting coil.

The strain sensitivity properties of critical current density in Nb_3Sn superconducting cables have been well established by many researchers. The relationship between the critical current density of strands and the uniaxial tensile strain at low temperature environment of 4.2 K has been measured by Ekin [13]. Based on the experimental results of Nb_3Sn tapes, a deviatoric strain model is presented to describe the relationship between the critical current density and the strain [14]. The scaling law critical strain rate proposed by Markiewicz et al. [15–18] has a clear physical meaning and the three-dimensional strain state of the superconductor is considered. In the last few years, some modified models have been proposed [19]. Thus, it is important to find the stress and strain in the strand and determine the degradation of the critical current. Since the filaments in the superconducting strand are usually twisted to reduce the AC loss [20,21], it is difficult to obtain the exact mechanical deformation in the strand with many twisted filaments. In addition, the theoretical and finite element methods have been proposed for the analysis of the stress and strain in the Nb_3Sn filaments [9,22–27]. The advantage of these methods is that the local stress and strain in the Nb_3Sn filaments can be obtained. For the composites with the twisted or curved filaments, some simplified models are also presented [28–32]. However, there are few 3D models which considered the effect of the twisted structure on the stress and strain in the superconducting strand. The Nb_3Sn strand contains a large number of filaments, which leads to the huge computation with the finite element method. Jing et al. established a theoretical model to investigate the superconducting strands with twisted filaments in three-dimensional space

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[5]. The mechanical response of strand under the thermal strain and applied strain was analyzed.

In this paper, we have made some extensions and studied the strain distributions in the strand deeply based on the Ref. [5]. Firstly, the Young's moduli and thermal expansion coefficients are dependent on the temperature. Then, it is necessary to consider the cooling process to determine the equivalent moduli. In addition, we adopt two stages to study the mechanical behavior in the strand. In the first stage, the filament bundle is equivalent to a uniform cylinder. The strain distributions of twisted filament bundles in the strand are investigated numerically. The two different models of Hitachi strand and EAS-TFAS strand are considered and the distributions of the strain in the filament bundles under the thermal strain and the applied strain are obtained.

2. Introduction to micromechanics theory

2.1. Mori-Tanaka method

The effective elastic properties of composites with several phases can be determined based on the micromechanics method. The existence of inclusion will lead to the disturbance of stress, which was studied by Eshelby with equivalent inclusion method [33,34]. Eshelby used the method of Green's function to analyze the eigenstrain problem. The strain in the ellipsoid can be given using the principle of superposition [33]

$$\varepsilon_{ij}^{(Inc.)} = \overline{\varepsilon_{ij}} + S_{ijkl}\varepsilon_{kl}^{*} \tag{1}$$

where $\mathbf{S}$ is Eshelby's tensor, $\mathbf{I}$ is the identity tensor, $\boldsymbol{\varepsilon}^*$ is the uniform eigenstrain, $\overline{\boldsymbol{\varepsilon}}$ and $\overline{\boldsymbol{\sigma}}$ represent the uniform strain and stress on the boundary, and C_{ijkl}^{0} is the stiffness coefficients of matrix, respectively. After substituting the constitutive laws of matrix and inclusion into the preceding formula, the eigenstrain can be written as

$$\boldsymbol{\varepsilon}^* = -(\delta\mathbf{CS} + \mathbf{C}^0)^{-1}\delta\mathbf{C}\overline{\boldsymbol{\varepsilon}} \tag{2}$$

Then, the strain in the inclusion can be given by

$$\boldsymbol{\varepsilon}^{(Inc.)} = [\mathbf{I} - \mathbf{S}(\delta\mathbf{CS} + \mathbf{C}^0)^{-1}\delta\mathbf{C}]\overline{\boldsymbol{\varepsilon}} = [\mathbf{I} + \mathbf{P}\delta\mathbf{C}]^{-1}\overline{\boldsymbol{\varepsilon}} \tag{3}$$

where $\mathbf{P} = \mathbf{SC}_0^{-1}$, $\delta\mathbf{C} = \mathbf{C}^{(Inc.)} - \mathbf{C}^0$, and $\mathbf{C}^{(Inc.)}$ stands for the stiffness coefficient of the inclusion. As the strain in the inclusion is obtained, the stress and strain distributions of homogenous solid can be determined.

The Mori–Tanaka method [35–38] was applied to consider the effect of inclusion on the stress in the matrix around it on the basis of Eshelby's equivalent inclusion method, and the Mori-Tanaka method assumes that the strain in the far field of the inclusion is the average strain in the matrix. The relationship between strain in the inclusion and strain in the matrix can be determined through the Eshelby's tensor. Further, the average strain in the rth inclusion can be expressed as

$$\boldsymbol{\varepsilon}_r = \mathbf{T}_r\left[c_0\mathbf{I} + \sum_{r=1}^{N_{Inc.}} c_r\mathbf{T}_r\right]^{-1}\overline{\boldsymbol{\varepsilon}} \tag{4}$$

where $\mathbf{T}_r = [\mathbf{I} + \mathbf{P}_r\delta\mathbf{C}_r]^{-1}$, c_0 and c_r are the volume fractions of the matrix and rth inclusion, and the expressions of the other variables are given in Ref. [5]. Furthermore, the effective moduli of composite can be obtained by the Mori–Tanaka method

$$\overline{\mathbf{C}} = \mathbf{C}_0 + \sum_{r=1}^{N_{Inc.}} c_r[(\mathbf{C}_r - \mathbf{C}_0)^{-1} + c_0\mathbf{P}_r]^{-1} \tag{5}$$

2.2. The equivalent model of filament bundle

Since the strand contains about ten thousand filaments, it is difficult to consider every filament which needs a large amount of computation. In order to simplify the calculation, the equivalent model is adopted and the filament bundle is assumed to be uniform and continuous, as shown in Fig. 1. It is to be noted that the material parameters are dependent on the temperature. The relationship between the parameters and temperature is given in Ref. [6], as shown in Fig. 2. The equivalent moduli and Poisson's ratios are obtained with the Mori-Tanaka method. The strand is

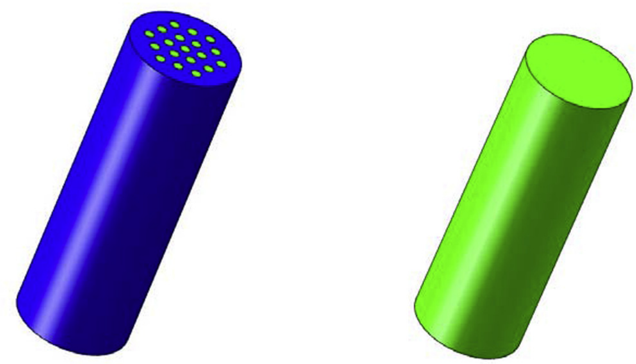

Fig. 1. The equivalent model of filament bundle.

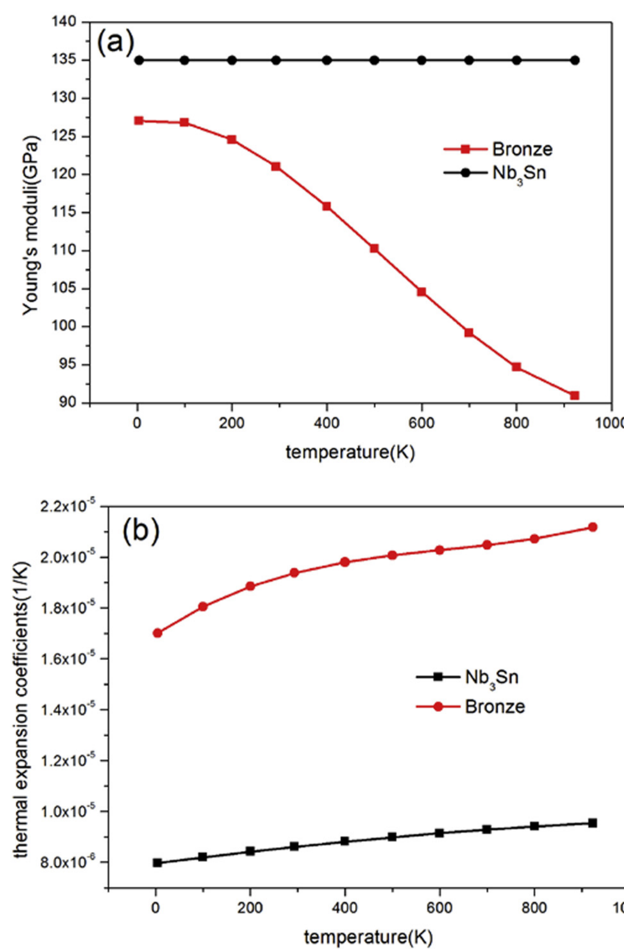

Fig. 2. The relationship between the parameters and temperature [6]: (a) Young's moduli and (b) thermal expansion coefficients.