Composite Structures 174 (2017) 366–374

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

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Thermal residual solutions of beams, plates and shells due to laminated object manufacturing with gradient cooling

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ARTICLE INFO

Article history:
Received 18 January 2017
Revised 12 March 2017
Accepted 23 April 2017
Available online 27 April 2017

Keywords:
Laminated object manufacturing
Gradient cooling
Thermal residual stress
Deformation
Analytical solution

ABSTRACT

Laminated object manufacturing (LOM) results in thermal residual stresses and deformations due to its material mismatch among layers and gradient cooling. The manufacturing and cooling processes in a laminated workpiece are decomposed into many turns. Through-thickness cooling gradients are considered in each layer forming turn. The thermal residual strains in each turn are deduced. The total thermal residual stresses are obtained by composing the strains in all turns. The workpieces with geometries of a beam, plate and shell are considered. For a beam or plate, the analytical solutions of the stresses and deformations in the axial and transverse directions are deduced separately and then synthesized to the total solutions. One-dimensional beams and two-dimensional plates have very similar solutions. For a symmetrical hollow cylinder or sphere, the analytical solutions of thermal residual stresses with through-thickness cooling gradients are also formulated. The numerical examples show that the assumption of synchronous cooling results in big errors and cooling gradients also induce thermal residual stresses and deformations. Four gradient cooling models are summarized according to various LOM manufacturing techniques and the thermal residual level is proportional to the cooling gradient. Processing sequence in functional gradient materials has a strong impact on thermal residual level.

1. Introduction

As one of additive manufacturing technologies, laminated object manufacturing (LOM) was ever defined in a narrow sense. A workpiece is formed layer on layer and behaves as a laminated structure. Additive manufacturing technologies are expanding continuously their family over the past decades. Hence, a generalized LOM may be defined here in a wider sense, including (1) classical LOM; (2) 3D printing; (3) functionally graded materials; (4) coatings on a substrate; etc. This generalized LOM has some common features in structures and thermoelastic responses. It needs high manufacturing temperature and/or uses more than one materials, which result in thermal residual phenomenon and reduce workpiece quality. Effective control of thermal residual phenomenon is a main goal to improve LOM techniques.

As far as the mechanisms to induce thermal residual phenomena, one is material mismatch among layers and the other is gradient cooling through layers. The studies on thermal residual phenomenon in LOM mainly focused on material mismatch among layers. Thermal residual stresses were induced by different contraction strains of layers when a workpiece cooled from the manufacturing temperature to the room temperature. It was assumed that all layers kept the manufacturing temperature during the whole forming process and cooled to the room temperature after the process at the same time. This assumption was defined as "synchronous cooling". It simplified the math formulation of analytical solutions of thermal residual stresses. According to this assumption, there was no thermal residual stresses if all layers used one material. However, this does not accord with facts. Actually the temperatures of most layers had already dropped below the manufacturing temperature before a whole workpiece was finished. A through-thickness temperature gradient had already existed and changed during the whole manufacturing process, which is defined as "gradient cooling" here. Hence, temperature gradients also result in thermal residual phenomena.

Since gradient cooling was not introduced, the almost all analytical studies on LOM thermal residual phenomenon were based on "synchronous cooling". The earliest study discussed a simple bi-beam [1] model and predicted only axial thermal residual strains and stresses. The stresses were over estimated because bending deformations were neglected. The further studies [2,3] considered bending deformations and improved accuracy. Some finite element analyses [4,5] were carried out. On the other hand the studies on

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http://dx.doi.org/10.1016/j.compstruct.2017.04.060
0263-8223/

mechanics of composite laminates have been carried out fully. The classical [6], first-order [7], higher-order [8,9] and high-order [10–12] laminate theories showed different accuracies when predicting mechanical responses of laminated structures. Their corresponding thermoelastic formulations [9,13] can be used to predict thermal residual phenomena in LOM workpieces with synchronous cooling or known temperature distributions. However, they cannot be used when a real manufacturing and cooling processes are considered, because cooling occurs and cooling gradients change during the whole manufacturing process. To the author's knowledge there is no publication on thermal residual phenomena in LOM with gradient cooling. On the other hand, the almost all studies on LOM thermal residual stresses were focused on beams. Few of them were extended to plates [14] and shells [15,16].

This work abandons synchronous cooling hypothesis and introduces "gradient cooling" to improve accuracy. The real manufacturing and cooling processes will be decomposed. The thermoelastic responses in each forming step will be analyzed and then the accumulative stresses will be obtained. Although cooling gradients occur along all three coordinates [16,17], the cooling gradient through layers is the main factor to induce thermal residual phenomena. In this work the thermal residual solutions of laminated object manufacturing with gradient cooling through thickness (layers) are deduced. A wide range of structures, including beams, plates and shells, are considered.

2. One-dimensional beams

A laminated beam has n layers with a thickness h and length l. Denote the top and bottom surfaces of the beam with $h/2$ and $-h/2$, respectively. Denote the top and bottom interfaces of the jth layer with z_j and z_{j-1}, respectively. E_j, ν_j and α_j are Young's modulus, Poisson ratio and thermal expansion coefficient of the jth layer, respectively. Denote ΔT the difference between the manufacturing and room temperatures. Both the axial and bending deformations are discussed below.

2.1. Axial deformations and stresses

LOM is an elevated temperature forming process. During the process each layer has different cooling time and so a temperature gradient through thickness occurs. All layers cool to the room temperature after the process. Hence, thermal residual phenomena occur in a dynamic and accumulative way. To characterize this phenomena the process of manufacturing and cooling must be decomposed into the following turns:

(1) Turn 1. When the first layer is finished and the forming of the second layer starts, the temperature of the first layer drops $\Delta t_1^{(1)}$ on average from the manufacturing temperature (Fig. 1(a)). The first layer has an axial thermal strain

$$\varepsilon^{(1)} = \alpha_1 \Delta t_1^{(1)} \tag{1}$$

The resulted axial thermal force of the first layer, $N_1^{(1)}$, is zero due to its free deformation.

(2) Turn 2. When the second layer is finished and the forming of the third layer starts (Fig. 1(b)), the temperatures of the first and second layers drop $\Delta t_1^{(2)}$ and $\Delta t_2^{(2)}$ from the manufacturing temperature, respectively. Denote the thermal strain in this turn with $\varepsilon^{(2)}$. One obtains the resulted axial thermal forces of the two layers

$$\begin{aligned} N_1^{(2)} &= E_1 h_1 (\varepsilon^{(2)} + \varepsilon^{(1)} - \alpha_1 \Delta t_1^{(2)}) \\ N_2^{(2)} &= E_2 h_2 (\varepsilon^{(2)} - \alpha_2 \Delta t_2^{(2)}) \end{aligned} \tag{2}$$

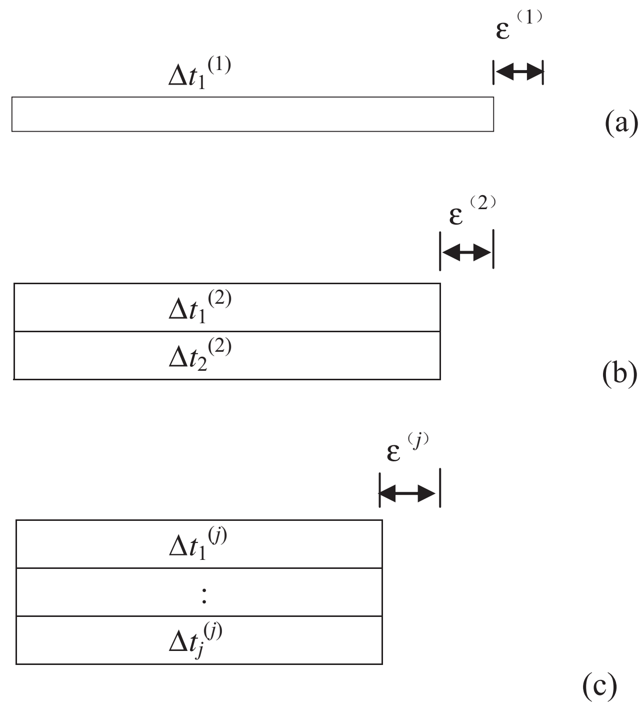

Fig. 1. Manufacturing and cooling processes.

The total force at the cross-section equals to zero, namely $N_1^{(2)} + N_2^{(2)} = 0$, so one obtains

$$\varepsilon^{(2)} = \frac{-E_1 h_1 \varepsilon^{(1)} + E_1 h_1 \alpha_1 \Delta t_1^{(2)} + E_2 h_2 \alpha_2 \Delta t_2^{(2)}}{E_1 h_1 + E_2 h_2} \tag{3}$$

(3) Turn j. The rest may be deduced by analogy. When the jth layer is finished and the forming of the $(j+1)$th layer starts (Fig. 1(c)), the temperature of the kth layer drops $\Delta t_k^{(j)}$ $(k = 1 \ldots j)$ from the manufacturing temperature. Denote the thermal strain in this turn with $\varepsilon^{(j)}$. The resulted thermal force in the kth layer is

$$N_k^{(j)} = E_k h_k \left(\sum_{m=k}^{j} \varepsilon^{(m)} - \alpha_k \Delta t_k^{(j)} \right) \tag{4}$$

The total force at the cross-section equals to zero, namely $\sum_{k=1}^{j} N_k^{(j)} = 0$, so one obtains

$$\varepsilon^{(j)} = \frac{-\sum_{k=1}^{j-1} E_k h_k \sum_{m=k}^{j-1} \varepsilon^{(m)} + \sum_{k=1}^{j} E_k h_k \alpha_k \Delta t_k^{(j)}}{\sum_{k=1}^{j} E_k h_k} \quad (j = 2, \ldots n-1) \tag{5}$$

(4) Turn n. After the whole process is finished, all layers cool to the room temperature. According to Eq. (5), the resulted thermal strain $\varepsilon^{(n)}$ in this turn is

$$\varepsilon^{(n)} = \frac{-\sum_{k=1}^{n-1} E_k h_k \sum_{m=k}^{n-1} \varepsilon^{(m)} + (\sum_{k=1}^{n} E_k h_k \alpha_k) \Delta T}{\sum_{k=1}^{n} E_k h_k} \tag{6}$$

Hence, the thermal residual strain $\varepsilon^{(j)}$ $(j = 1 \ldots n)$ in each turn may be calculated according to Eqs. (1), (5) and (6). Finally the accumulative axial strain in the jth layer equals to $\sum_{k=j}^{n} \varepsilon^{(k)}$ and the axial thermal residual stress in the jth layer equals to

$$\sigma_{jN} = E_j \left(\sum_{k=j}^{n} \varepsilon^{(k)} - \alpha_j \Delta T \right) \tag{7}$$