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Free vibration analysis of angle-ply symmetric laminated plates with free boundary conditions by the discrete singular convolution

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ABSTRACT

The discrete singular convolution (DSC) is used for free vibration analysis of three-layer angle-ply symmetric laminated plates with free boundaries, including laminated plates with two adjacent free edges. During formulating the weighting coefficients of derivatives having different orders, two Taylor series expansions with different orders are used to eliminate the degrees of freedom at fictitious points outside the physical domain. Thus, the difficulty in handling free boundary conditions by using the DSC is overcome. Results are presented and compared with either exact solutions or the ones obtained by the differential quadrature method (DQM). It is shown that the DSC with the novel way to apply the boundary conditions yields very accurate lower mode frequencies as well as relatively accurate higher mode frequencies. The excellent performance of the DSC on high mode frequencies of beams and isotropic plates is retained and also independent of boundary conditions.

1. Introduction

The demand in the design of high performance air vehicles has resulted in consideration of fiber-reinforced composite materials as the structural materials to improve the structural efficiency [1]. Plates and shells are common structural elements. Their static, buckling and vibration behaviors are of important to the designers and thus have been received great attentions [2–5]. Due to the anisotropic nature of the fiber-reinforced composite materials, analytical solutions are rarely available even for plates with simple boundary conditions [3–8], therefore, the finite element method (FEM) is the major approach in practice.

Besides the well-known finite element method, various approximate and numerical methods, such as Rayleigh-Ritz method [9,10] and Levy method [11], the differential quadrature method (DQM) [12–18], the meshless method [19], and the discrete singular convolution (DSC) [20–31], have also been employed for obtaining solutions of anisotropic as well as composite structures.

It has been shown that the DQM is one of the alternative efficient methods for analyzing anisotropic rectangular plates [12–18]. Due to its compactness and computational efficiency [14,15], the DQM is more attractive than the Rayleigh-Ritz and Levy methods for analyzing anisotropic composite plates. Now the DQM has been well-developed and can yield numerically exact solutions in many cases. Thus the DQM has become a powerful new technique for analyzing composite structures [13]. Similar to many other numerical methods, however, the DQM cannot yield accurate higher mode frequencies. Therefore, the research on other alternative efficient method, which can yield not only accurate lower mode frequencies but also relatively accurate higher mode frequencies, is an ongoing activity.

The discrete singular convolution, proposed by Wei [32,33], is such an efficient method. It has been demonstrated that the DSC can yield not only accurate lower mode frequencies but also relatively accurate higher mode frequencies for beams and isotropic rectangular plates [34–38]. The DSC has been successfully used to analyze the mechanical behavior of laminated composite structures [22–31]. However, most cases investigated by the DSC are rectangular plates or shells without a free boundary. The free vibration of laminated rectangular plates with free boundaries was investigated by using the DSC [27]. Perhaps due to the difficulty in treating the boundary conditions at the free corners by the DSC with Taylor series expansion to eliminate the degrees of freedom at fictitious points outside the plate, the results presented in [27] are not very accurate. This indicates that the title problem has not been completely solved yet and deserves further investigations. How to implement boundary conditions properly is a very important issue for success by using various strong form numerical methods.

Currently, several ways are available in applying various boundary conditions by using the DSC. The method of symmetric extension is used for applying the fixed boundary conditions and the

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method of anti-symmetric extension is used for applying the simply supported boundary conditions [33]. The iteratively matched boundary (IMB) method [39] as well as the matched interface and boundary (MIB) method [40] are mainly used for applying the free boundary conditions. Although the IMB or MIB can be also used to apply other boundary conditions, however, the methods of anti-symmetric or symmetric extension are more convenient than the IMB or MIB for applying the simply supported or clamped boundary conditions. The method of Taylor series expansion [27,28,38], completely different from all above-mentioned methods, eliminates the degrees of freedom (DOFs) at fictitious points outside the physical domain without using the boundary conditions and thus is a general method to apply any kind of boundary conditions rigorously.

Very recently, it is reported that if the boundary conditions are appropriately applied, the DSC can yield not only accurate lower mode frequencies but also relatively accurate higher mode frequencies for beams and isotropic rectangular plates with any combinations of boundary conditions [38]. To the best of author's knowledge, however, the performance of the DSC on the higher mode frequencies of laminated composite plates has not been reported thus far, especially for laminated plates with two adjacent free edges.

The objective of this investigation is to extend the DSC for the free vibration analysis of anisotropic rectangular plates with two adjacent free edges. Two Taylor series expansions with different orders are used to eliminate the DOFs at fictitious points outside the plate during formulations of weighting coefficients of derivatives with different orders. Additional derivative degrees of freedom at boundary points are introduced for applying various boundary conditions rigorously. For verifications, DSC results are compared with exact solutions, finite element data and results obtained by the differential quadrature method (DQM). Three-layer angle-ply symmetric laminated square plates with seven combinations of boundary conditions and several ply angles are investigated to show the effect of the boundary condition and the material anisotropy on the frequencies as well as on the rate of convergence of the DSC. The performance of the DSC on both lower and higher mode frequencies of anisotropic square plates with two adjacent free edges is studied.

2. Discrete singular convolution

For completeness, the DSC in one dimension is briefly reviewed first. For two-dimensional problems, the approximation displacement field can be assumed by using a tensor product of the displacement function in one-dimension.

Denote N the total number of grid points, $\Delta = L_c/(N-1)$, where L_c equals to the plate length a or plate width b in this paper. The N grid points are denoted by $x_i = (i-1)\Delta (i = 1, 2, \ldots, N)$. In the DSC, the n^{th} (n = 0,1,2...) order derivative of a function $w(x)$ is approximated via a discredited convolution as [32–37]

$$w^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_{\alpha,\sigma}^{(n)}(x - x_k) w(x_k) \tag{1}$$

where $2M+1$ is called the computational bandwidth, $x_k = k\Delta$ $(k = -M, \ldots, -1, 0, 1, \ldots, M)$ are uniformly distributed grid points, $\delta_{\alpha,\sigma}(x - x_k)$ is a collection symbol for the DSC kernels and its n^{th} order derivative is given by

$$\delta_{\alpha,\sigma}^{(n)}(x - x_k) = \left(\frac{d}{dx}\right)^n \delta_{\alpha,\sigma}(x - x_k) \tag{2}$$

In the DSC algorithm, several delta kernels are available [37]. The two widely used delta kernels are the regularized Shannon kernel and the non-regularized Lagrange's delta sequence kernel. The DSC with the regularized Shannon kernel is commonly called the DSC-RSK [23–37] and the DSC with the non-regularized Lagrange's delta sequence kernel is commonly called the DSC-LK [27,28,34,38].

The regularized Shannon kernel is given by [37]

$$\delta_{\alpha,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right] \tag{3}$$

where $\alpha = \pi/\Delta$, σ is the controllable parameter which should be carefully selected. The criterion to select the controllable parameter σ with a given Δ and required accuracy is given by Wei et al. in [37].

The non-regularized Lagrange's delta sequence kernel is given by

$$\delta_{\alpha,\sigma}(x - x_k) = \begin{cases} L_{M,k}(x - x_k) & \text{for } -\beta \leqslant x \leqslant \beta \\ 0 & \text{otherwise} \end{cases} \text{for } M = 1, 2, \ldots \tag{4}$$

where $\beta \leqslant L_c$ and $L_{M,k}(x)$ is the Lagrange interpolation defined by [35]

$$L_{M,k}(x - x_k) = \prod_{j=k-M, j\neq k}^{k+M} \frac{x - x_j}{x_k - x_j} \quad (M \geqslant 1) \tag{5}$$

The differentiation in Eq. (2) can be easily carried out for the two kernels. For the DSC-RSK, analytical expressions are used to compute its various derivatives at a grid point and thus the number of grid points can be very large [38]. For the DSC-LK, the kernel is already discretized and the explicit formulas to compute the weighting coefficients existing in the DQM [18,28] can be employed to obtain the derivatives at a grid point.

The DSC-LK does not contain a controllable parameter σ and thus is simpler than the DSC-RSK. Besides, previous research showed that the DSC-LK can yield similar solution accuracy as the DSC-RSK [38]. Therefore, the DSC-LK is used in present investigations for simplicity. Besides, the DSC-LK is simply called the DSC since only one delta kernel is used in this paper.

A fourth-order partial differential equation is to be solved by using the DSC and the highest value of n in Eq. (2) is four. Take point 1 as an example, the first- to the fourth-order derivatives at x_1 = 0 are given by,

$$w^{(1)}(0) = \sum_{j=-M}^{M} \tilde{A}_{0j} w(x_j) = \sum_{j=-M}^{M} \tilde{A}_{0j} w_j \text{ or } w^{(1)}(x_1) = \sum_{j=-M}^{M} \tilde{A}_{1j} w_j \tag{6}$$

$$w^{(2)}(0) = \sum_{j=-M}^{M} \tilde{B}_{0j} w(x_j) = \sum_{j=-M}^{M} \tilde{B}_{0j} w_j \text{ or } w^{(2)}(x_1) = \sum_{j=-M}^{M} \tilde{B}_{1j} w_j \tag{7}$$

$$w^{(3)}(0) = \sum_{j=-M}^{M} \tilde{C}_{0j} w(x_j) = \sum_{j=-M}^{M} \tilde{C}_{0j} w_j \text{ or } w^{(3)}(x_1) = \sum_{j=-M}^{M} \tilde{C}_{1j} w_j \tag{8}$$

$$w^{(4)}(0) = \sum_{j=-M}^{M} \tilde{D}_{0j} w(x_k) = \sum_{j=-M}^{M} \tilde{D}_{0j} w_j \text{ or } w^{(4)}(x_1) = \sum_{j=-M}^{M} \tilde{D}_{1j} w_j \tag{9}$$

where $\tilde{A}_{1j}$, $\tilde{B}_{1j}$, $\tilde{C}_{1j}$ and $\tilde{D}_{1j}$ are called the weighting coefficients of the first-, second-, third- and fourth-order derivatives at x_1 = 0.

The weighting coefficients $\tilde{A}_{0k}$, $\tilde{B}_{0k}$, $\tilde{C}_{0k}$ and $\tilde{D}_{0k}$ can be computed explicitly by [18,28],

$$\tilde{A}_{0j} = \frac{\prod_{m=-M, m\neq 0,j}^{M} (-x_j)}{\prod_{m=-M, m\neq j}^{M} (x_j - x_m)} (j = -M, \ldots, -1, 1, \ldots, M), \ \tilde{A}_{00} = \sum_{\substack{m=-M \\ m\neq 0}}^{M} \frac{1}{(-x_m)} \tag{10}$$