Composite Structures 170 (2017) 135–145

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Fundamental frequency maximization of orthotropic shells using a free-form optimization method

Jin-Xing Shi [a], Tomohiro Nagano [b], Masatoshi Shimoda [a,*]

[a] Department of Advanced Science and Technology, Toyota Technological Institute, 2-12-1 Hisakata, Tenpaku-ku, Nagoya, Aichi 468-8511, Japan
[b] Graduate School of Advanced Science and Technology, Toyota Technological Institute, 2-12-1 Hisakata, Tenpaku-ku, Nagoya, Aichi 468-8511, Japan

ARTICLE INFO

Article history:
Received 6 May 2016
Revised 24 January 2017
Accepted 1 March 2017
Available online 6 March 2017

Keywords:
Free-form optimization
Fundamental frequency maximization
Gradient method
Orthotropic shell
Repeated eigenvalues

ABSTRACT

In this study, a free-form optimization method is proposed that maximizes the fundamental frequencies of the orthotropic shells to avoid vibrational resonance. The negative fundamental vibrational eigenvalue is employed as the objective function, which is minimized by subjecting to the governing equation of the natural frequency analysis and area constraint. In the free-form optimization process, the natural frequency analysis of the orthotropic shells is performed to calculate the shape gradient function. The shape gradient function is then applied to the velocity analysis for determining the optimal shape variation. The repeated eigenvalues are considered by converting the fundamental eigenvalue to a summation form of the repeated eigenvalues. The proposed optimization method is validated using three examples of the orthotropic shells. The numerical results show that the optimized shapes of the orthotropic shells are smooth, and their fundamental frequencies are significantly enhanced using the proposed free-form optimization method.

1. Introduction

Because of their advantages, such as low weight, high strength, and anisotropic-material characteristics, composite shells considering material orientations have been widely used as structural members in aircraft, marine, automotive, and civil engineering [1]. Orthotropic shells, which are a subset of anisotropic shells, have different material properties along two mutually orthogonal directions. Since the last few decades, theoretical analyses have been conducted on anisotropic shells considering their mechanical properties such as stiffness [2,3], buckling [3–6], vibration [3–5,7], and wave propagation [8]. With respect to the vibration of the shells, Liew et al. [9] summarized most of the studies conducted on the isotropic shells since the 1970 s based on the Kirchhoff–Love, Reissner–Mindlin, and higher-order shell theories. Considering the material orientations, Qatu et al. [10] reviewed most of the studies on the dynamic analysis of the anisotropic shells conducted during 2000–2009. They noted that the finite element method (FEM) was increasingly applied to analyze the vibration of the anisotropic shells having various geometries. In this study, the focus is on optimizing the shapes of the orthotropic shells for maximizing the natural frequency using the FEM and a gradient method for the shape optimization of shells [11].

To obtain the best mechanical performance, some design optimization studies were conducted on the anisotropic shells considering low weight (or thickness) [12,13], stiffness [14–19], buckling [20–23], and dynamic problems [14,20,24–30]. Among these studies, only a few concentrated on the orthotropic shells [14,15,25]. For instance, Luo and Gea [14] proposed a bending equivalent orthotropic model to investigate the optimal bead orientation of the shell structures for both static and dynamic cases based on the stress resultant-strain and stress couple-curvature relations of the bead-stamped shell elements. Based on the multiquadric method and an optimization technique, Roque et al. [15] analyzed the orthotropic shells for the static cases using the mesh-less method with the help of a higher-order shear deformation theory. By employing the non-gradient evolutionary genetic algorithm, Cho [25] successfully optimized the local fiber angles of the orthotropic shells subjected to a hygrothermal environment in order to minimize the dynamic responses.

In the design optimization of the structures, the dynamic problem includes both the natural vibration problem [14,20,27–30] and the frequency response problem [25,26,31]. The fundamental frequency maximization problem considered in this study is classified under the natural vibration problem, as it involves eliminating the vibrational resonance. Nshanian and Pappas [20] formulated a mathematical programming problem using the segment ply angles and thicknesses as the design variables and applied it for maximizing the fundamental natural frequency of the anisotropic shells.

* Corresponding author.
E-mail address: shimoda@toyota-ti.ac.jp (M. Shimoda).

http://dx.doi.org/10.1016/j.compstruct.2017.03.007
0263-8223/

Nomenclature

$(\bullet)^{(m)}$ $(m = 1, 2, 3 \ldots)$ mth eigenvector of eigenvalue
$(\bar{\bullet})$ variation
$(\bullet)' = (\dot{\bullet}) - (\bullet)_{,i} V_i$ shape derivative
$(\dot{\bullet})$ material derivative
$(\bullet)_{,i} = \partial(\bullet)/\partial x_i$ partial differential notation
$a(\cdot,\cdot)$ virtual work in terms of rigidity
A mid-surface of the orthotropic shell
A_s mid-surface of the orthotropic shell after shape variation
$b(\cdot,\cdot)$ virtual work in terms of inertia
c side length of the shell
C_Θ admissible function space satisfying the constraints of the shape variation
D bending rigidity
E_α $(\alpha = 1, 2)$ orthotropic Young's modulus
$\{E_{\alpha\beta\gamma\delta}\}$ $(\alpha, \beta, \gamma, \delta = 1, 2)$ orthotropic elastic tensor with respect to the membrane force
$\left\{E^S_{\alpha\beta}\right\}$ $(\alpha, \beta = 1, 2)$ orthotropic elastic tensor with respect to the shear force
$\boldsymbol{G} = G\boldsymbol{n}$ shape gradient function
G_V, G_S shape gradient density functions
$H^1(A)$ Sobolev space of order 1
k shear correction factor
L Lagrangian function
$\boldsymbol{n}$ normal vector
$\boldsymbol{n}^{btm}$ normal vector at the bottom surface of the shell
$\boldsymbol{n}^{mid}$ normal vector at the mid-surface of the shell
$\boldsymbol{n}^{top}$ normal vector at the top surface of the shell
s iteration history of the shape variation
S area of the orthotropic shell
S_0 initial area of the orthotropic shell
$\hat{S}$ constraint area of the orthotropic shell
t thickness of the shell
$\boldsymbol{u} = \{u_i\}$ $(i = 1, 2, 3)$ displacement vector in the local coordinate system
$\boldsymbol{u}_0 = \{u_{0\alpha}\}$ $(\alpha = 1, 2)$ in-plane displacement vector in the local coordinate system
U admissible function space satisfying the Dirichlet boundary conditions
$\boldsymbol{V} = \{V_n\}$ $(n = 1, 2, 3)$ design velocity field
$\boldsymbol{V}_0 = \{V_{0\alpha}\}$ $(\alpha = 1, 2)$ in-plane design velocity field
V_3 out-of-plane design velocity field
w out-of-plane displacement
(x_1, x_2, x_3) local coordinate system
(X_1, X_2, X_3) global coordinate system
$\boldsymbol{X}$ position vector in the global coordinate system
$\boldsymbol{X}_s$ position vector in the global coordinate system after the shape variation
∂A boundary of A
$\mathbb{R}$ a set of real numbers
α spring constant of the Robin boundary condition
δ tolerance for determining the repeated eigenvalues
ε a small positive number
κ twice the mean curvature of the orthotropic shell
$\lambda^{(m)}$ $(m = 1, 2, 3 \ldots)$ mth vibrational eigenvalue
μ shear modulus of the isotropic shell
μ_α $(\alpha = 1, 2)$ shear modulus of the orthotropic shell
ν Poisson's ratio
$\boldsymbol{\theta} = \{\theta_\alpha\}(\alpha = 1, 2)$ rotation angle vector in the local coordinate system
$\boldsymbol{\theta} = \{\theta_\alpha\}(\alpha = 1, 2, 3)$ rotation angle vector in the global coordinate system
ρ density
ω angular frequency
$\bar{\omega}$ non-dimensional frequency parameter
Δs small amount of the shape variation
Λ Lagrange multiplier of the area constraint
Ω domain of the orthotropic shell including the thickness
Ω_s domain of the orthotropic shell after the shape variation

The results showed that efficient shells were obtained using the optimal variable ply angle configurations. Furthermore, recently, several studies were conducted on maximizing the fundamental frequency of the anisotropic shells [27–30]. Using the lamination parameters as the design variables, Trias et al. [27] maximized the fundamental frequency of the anisotropic cylinders for a large number of thicknesses and aspect-ratio combinations. For simultaneously maximizing fundamental natural frequencies of the anisotropic plates, Vosoughi et al. [28,29] introduced a hybrid multi-objective optimization technique combined with the differential quadrature method, undominated sorting genetic algorithm II, and Young's bargaining model. Hu and Chen [30] maximized the fundamental frequencies of the anisotropic truncated conical shells under the axial compressive forces using the golden section method.

The aforementioned studies related to the fundamental frequency maximization problem exhibited their applicability and usefulness in design optimization of the anisotropic shells. However, most studies adopted the parametric optimization methods in the vector space belonging to the discrete system that use the design parameters (dimensions, thicknesses, material orientations, etc.), for which the designers need to have considerable knowledge and experience. Compared to the parametric optimization method, the non-parametric (or parameter-free) free-form optimization techniques based on the gradient method in the function space (belonging to the continuous system) has an advantage of treating all nodes as design variables in the large-scale problem. Thus, the method does not require the design parameters in advance. In the previous studies conducted by the authors, a free-form optimization method based on the H^1 gradient method was developed for designing the composite solids [32–34], damping material-inlaid plates [35], and isotropic shells [36,37]. In these previous studies, we employed the free-form optimization method to optimize the shapes of the solids and shells/plates considering the stiffness, structural-acoustic coupling, and buckling problems. All of these studies focused on the shape optimization of isotropic materials; however, shape optimization of anisotropic materials is also a worthwhile work, and we have employed the optimization method to maximize the stiffness of the orthotropic shells [16]. In this study, the objective is to extend this method to optimize the shapes of the orthotropic shells for maximizing the fundamental frequency considering the repeated eigenvalues. In the upcoming study, we will optimize the shapes of the anisotropic shells considering the variation in the material orientation of the anisotropic shells. As such, in this study, the first step is to optimize the shape of the orthotropic shells without considering the variation in the material orientation.

The free-form optimization method employed in this study has four essential steps: (I) Deriving the shape gradient function based on the Lagrange multiplier method. (II) Calculating the shape gradient function based on the results of the natural frequency analysis. (III) Using the shape gradient function to determine the optimal shape variation (i.e., optimal design velocity) based on the H^1 gradient method. (IV) Modifying the shape using the optimal design