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A hierarchical finite element for composite laminated beams using a refined zigzag theory

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ABSTRACT

In this work a kinematics for laminated beams enriched with a refined formulation ZigZag (RZT), originally presented by Tessler et al. in 2007, introduced in a hierarchical one dimensional type "p" finite element is presented. The finite element employs Lagrange polynomials for the approximation of the degrees of freedom of the ends (nodes) and orthogonal Gram-Schmidt polynomials to the internal degrees of freedoms. This finite element allows a very low discretization, is free of shear locking and behaves very well when the analysis of laminated composites with accurate determination of local stresses and strains at laminar level is necessary.

This element has been validated in the analysis of laminated beams with various sequences of symmetric and asymmetric stacking, studying in each case its accuracy and stability.
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1. Introduction and review

Laminated composite beams are basic components for several structural engineering applications, due to their excellent mechanical properties, namely high specific strength and stiffness, long fatigue life, wear resistance and enhanced design freedom on a micro- and macro-mechanical level. The behavior of laminated beams is governed by a wide number of parameters due to their complex behavior. Moreover, specific problems arise such as delamination and complex damage and failure mechanisms that need a proper modeling for an accurate appraisal of the study of their mechanics. In particular, as it is shown by Carrera [1–2], a slope discontinuity on the displacement field occurs at the interface between two perfectly bonded layers because of the transverse anisotropy, i.e. the difference in layer-wise transverse shear and normal moduli. This is known as the ZigZag (ZZ) phenomenon.

Considering theses aspects a number of theories have been proposed for the analysis of composite laminates. Theories used for the through –thickness variation of the state variables (unknowns are of displacement type) can be classified as: equivalent single layer models (ESL), layer-wise models (LW) and zigzag models (ZZ). The well-described unified formulation, initially presented by Carrera [3] and extended by Demasi [4–8], describes precisely and clearly the models, types and class of these theories.

In the ESL theories the assumed displacements vary continuously across the laminate thickness and the number of unknowns is independent of the number of layers. ESL models include mainly three major categories, i.e., the classical theory (CT), the first-order theory (FDT), and the higher-order theory (HOT). The CT known as Euler–Bernoulli beam theory is the simplest one and is inaccurate for reasonably thick laminated beams and/or for highly anisotropic composite beams. The inaccuracy is due to neglecting the transverse shear strains in the laminate. The FDT by Timoshenko [9] considers constant transverse shear strain distribution through the beam thickness and, thus, a shear correction factor has to be incorporated to adjust the transverse shear stiffness. The accuracy of FDT solutions depend on the shear correction factor which cannot in general be determined a priori apart from very special cases [10]. Moreover, FDT produces piecewise constant transverse shear stresses that violate the interlaminar continuity (IC) conditions and the traction-free conditions at the top and bottom surfaces. To overcome these shortcomings and to avoid the use of shear correction factors, a number of high-order theories with different shear strain shape functions were introduced. In general, the cross section is allowed to deform in any form by including higher order terms in the axiomatic expansion of the displacement field along

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the beam direction (x-axis) as a suitable smooth function of transverse direction (z-axis). In this sense different shape functions have been proposed such as polynomial [11–14], trigonometric [15–18], exponential [19–21] and hyperbolic functions [22,23]. Carrera et al. [24] discussed a number of refined beam theories which were obtained expanding the unknown displacement variables over the beam section axes by adopting Taylor's polynomials, trigonometric series, exponential, hyperbolic and zigzag functions, by using the Unified Formulation introduced by Carrera [3]. A class of theories often included into HOT are the advanced higher order theories, denoted as AHOT, where transverse normal strains are incorporated by extending the expansion of the transverse displacement. For instance, Vidal et al. [25] proposed the approximation of the displacement field as a sum of separated functions of axial and transverse coordinate by adopting the Proper Generalized Decomposition procedure. HOT gives a continuous variation of the transverse shear strain across the thickness but shows discontinuity in the shear stress distribution at the layer interfaces (if they are computed through the constitutive equations) due to different values of shear rigidity at the adjacent layers. But the actual behavior of a composite laminate is opposite i.e., the transverse shear stress must be continuous at the layer interface and the corresponding strain may be discontinuous [26].

In LW models [27–34] the displacement field within each layer is prescribed and compatibility conditions are applied between adjacent layers in the laminate to recover the model of the laminate as a whole. These models provide realistic descriptions of kinematics at the ply level and they have the capacity to take into account the zigzag effect. However, LW approaches suffer from an excessive number of displacement variables in proportion to the number of layers and hence they are too expensive in terms of computational cost and hardly appropriate for practical applications.

ZigZag models include a set of layer independent theories in which a LW discontinuous function is a priori selected to enrich the kinematical model in such way that the interface conditions are met. So, in these theories, the in-plane displacements have piece-wise variation across the beam thickness and the number of unknowns results independent of the number of layer. Examples of ZZ theories are those found in articles published by Murakami [35], Lee et al. [36], Cho and Paramerter [37], Cho and Averill [38], Vidal and Polit [39,40].

The research activity about the modeling of laminated structural members and the corresponding analytical or numerical solutions are numerous. In particular, as this paper is devoted to ZigZag models a complete and extensive assessment about the subject can be found in Carrera [1]. Other reference in the topic is the review paper by Chakrabarti et al. [26]. On the other side, Groh and Weaver [41] present, in the article introduction, a comprehensive overview of the different theories that are used for the analysis of highly heterogeneous laminated beams.

Many ZigZag theories requires C^1 continuity for the deflection field, which is a drawback versus simpler C^0 continuous FEM approximations [42]. Tessler et al. [43–45] developed a refined zigzag theory (RZT) that allows the use of C^0 continuous interpolation for all the kinematic variables. The kinematics of RZT is essentially that of FDT enhanced by a zigzag field which has the property of vanishing on the top and bottom surface of the laminate.

Along with the development of beam theories, there has been significant development towards the solution methodologies. Analytical solutions are applicable for a few particular classes of beam configuration [46,47]. The development of computational technologies makes it quite possible to implement numerical methods for the practical applications. Among these, FEM is most popular and versatile method for investigating the structural behavior of arbitrary shaped components. In this context Oñate et al. [42] developed a simple 2-noded beam element based on the RZT theory, where shear locking is avoided using reduced integration on selected terms of the shear stiffness matrix. The classical version or h version of FEM was used in this paper, where the accuracy of the solution is achieved by refinement of finite element mesh.

Unlike the h FEM version, in the p version of FEM the mesh remains constant while the degree of the interpolation polynomial is gradually increased to the desired accuracy [48]. The degrees of freedom of a p element are constituted by the degrees of freedom of the one-dimensional element ends (nodes) and the amplitudes of the shape functions within the element. The p-version is characterized by being more robust than the version h [49], in other words the performance of the p-version is much less sensitive to input data tan the h-version. For example, the p version allows proper treatment of elements with high slenderness, as it is free of shear locking. This is especially important in the analysis of laminated composite beams, where a more rigorous stress analysis at laminar and inter-laminar level is necessary. Several demonstrative examples and theoretical proofs of the advantages of the p-version FEM can be found in the literature [48,50–54]. Recall that the advantages of the p version are not limited to the greater convergence rate. In fact, with h methods, the accuracy of the solution is determined by executing several analyses with different meshes, an expensive and time-consuming process, both because of the computational cost and because of the operator time required to define the different models. In p-convergent approximations, the number of finite elements is determined by the geometry and is small [55].

In this paper a hierarchical one-dimensional finite element, based on the ZigZag refined theory by Tessler et al. [43–45] is proposed. This finite element has two end nodes and four degrees of freedom per node. To approximate the kinematics variables of this formulation Lagrange polynomials as local support functions are used, and orthogonal polynomials generated by means of recurrence Gram-Schmidt expressions [58–59] are employed as functions of hierarchical enrichment [60–63]. It is necessary to emphasize here that one of the main novelty of the proposed model is the obtaining of a hierarchical finite element within the framework of a Zig-Zag theory, considering local support functions of C_0 type and achieving a robust finite element free of shear locking. Besides, the developed formulation is appropriated for the analysis of symmetric and non-symmetric laminated beams in a general and unified way, since all mechanical coupling are considered. Another important and salient feature of the developed model is the capacity it has for its application to the delamination study as will be seen in Section 8.

The proposed finite element has been computationally implemented. To verify the results, the order of the approximation can be selectively increased. This operation is carried out very efficiently because it is not necessary to generate a new mesh and because the new linear stiffness matrix contains the preceding one. It is demonstrated that the proposed hierarchical finite element is free of shear locking and, in order to assess its accuracy and stability, it has been applied to the analysis of laminated beams with symmetrical and non-symmetrical stacking sequence with different boundary conditions.

2. Formulation of the mechanical problem

Let us consider a laminated beam of total thickness h and length L as shown in Fig. 1. The Cartesian coordinate system (x, y, z) is taken such that the $x - y$ plane $(z = 0)$ coincides with the midplane of the beam, the y axis is along the width (b) of the beam; resulting in a beam domain

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