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Non-classical vibration behavior of highly anisotropic corrugated laminates

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ABSTRACT

This paper presents a detailed investigation about the vibration behavior of corrugated laminates. In highly anisotropic corrugated laminates different non-classical vibration modes were observed and are reported in this work. Apart from in-plane modes, we show in particular shear rotational modes which occur due to the high anisotropy, the distribution of mass, and the influence of the shear compliance. The work contains a detailed FEM study, a comparison with an equivalent plate model and an analytical model and examines the limitations of the latter two. It points out for which geometry and material parameters the well known and often used homogenized plate models are applicable. Parametric studies are conducted investigating the influence of the corrugation amplitude, the aspect ratio, the anisotropy of the material, and boundary conditions on the vibration behavior. The found results can be used for the design of highly anisotropic corrugated laminated plates and the analysis of their vibration behavior.

1. Introduction

Corrugated laminates have a very high anisotropy, not only because of the anisotropic laminated materials, but mainly due to geometric reasons. This makes them interesting as structural elements in various applications [1]. They have been used since decades as stability elements, e.g. in Junkers *Ju-52* [2] and are widely used as sandwich cores [3,4]. Lately, they have been suggested as ideal candidates for morphing skins [5,6], for example in morphing wing applications [7,8]. In morphing wings flaps and slats are replaced by shape adaptive wings which requires flexible elements such as corrugated laminates that provide axial compliance in order to keep actuation forces low while providing high bending stiffness to withstand external forces. Morphing wing solutions have been suggested as being more efficient than conventional wings [9].

In order to reduce numerical costs, many models have been developed to find the equivalent stiffness properties of corrugated laminates. Xia et al. suggested an analytical model for symmetric and balanced laminates [10,11]. Kress et al. proposed an analytical model that is valid for corrugations consisting of circular sections [12].

Corrugated laminates with high amplitudes can be favorable due to their stress distribution [13], and enhanced anisotropic behavior. This high anisotropy can lead to in-plane eigenmodes of natural vibrations. Wang et al. [14] also reported in-plane modes in highly anisotropic sandwich structures where the shear compliance is of importance. While the out-of-plane vibration behavior of orthotropic plates is already understood well [15], not much literature about in-plane vibration of highly anisotropic plates is available. Bardell et al. [16] analyzed free in-plane vibrations in isotropic plates. Semenyuk et al. [17] investigated different vibration modes of corrugated cylindrical shells. Liu et al. [18] developed an analytical model to analyze in-plane vibration modes for orthotropic rectangular plates. However, non-classical vibration modes of corrugated laminates with extremely high anisotropy – where in comparison to flat plates also through-thickness mass distribution has to be accounted for – have not been reported so far in literature. These non-classical vibration modes also limit the valid solution space of often used equivalent plate models. To the authors' knowledge also the question of how anisotropy due to geometry and material influence the observed mode shapes and frequencies of corrugated laminates, which is of high importance for design aspects, have not been addressed so far.

In the present paper we investigate the influence of anisotropy on the vibration behavior of corrugated laminates. We use a detailed FEM model to investigate the vibration behavior as a function of anisotropy due to geometry. Besides the intuitive out-of-plane mode known from anisotropic plate vibration models, we also report in-plane modes and non-classical shear rotational modes. We compare these results with a numerical and an

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analytical homogenized plate model which are very efficient in terms of numerical costs. Both models use equivalent stiffness properties as input parameters. Comparing the three models allows to quantify the valid solution space for the widely used homogenized plate models which are not able to capture all the observed inertia effects. Furthermore, we investigate the interaction of anisotropy due to material and geometry and their influence on corrugated laminates vibration behavior using analytical modeling. In a parametric study the main influencing parameters such as corrugation amplitude, plate aspect ratio, and material properties are examined concerning eigenmodes and -frequencies.

The following section of the paper presents the detailed FEM shell model, an equivalent homogenized plate FEM model, and an analytical homogenized model. Then the characteristically observed eigenmodes are presented depending on the anisotropy due to geometry. These include out-of-plane, in-plane and rotational modes. The subsequent section investigates the valid solution space for homogenized modeling approaches. Then we add a further complexity by considering the anisotropy due to material and investigate the influence on the vibration modes. Furthermore, a parametric study is presented examining the influence of geometry, material, and aspect ratio on the eigenmodes and frequencies. The paper closes with a discussion of the results and a conclusion.

2. Modeling approach

For the modeling of the eigenvalues three different approaches are used:

- an equivalent plate model: efficient and widely used for modeling of corrugated laminates, valid for arbitrary boundary conditions,
- a finite element shell model: allows to accurately map the geometry, valid for arbitrary boundary conditions, but more costly than the other models, and
- an analytical model: very efficient, but only valid for simply-supported boundary conditions.

Using three different models allows to compare the different results to each other and can be used as a verification in case that all models account for the same assumptions and simplifications. The following subsections describe these models.

2.0.1. Geometry definition

Fig. 1 shows a unit-cell of a corrugated structure consisting of circular sections as we use it in this work. The corrugation is defined by a corrugation amplitude c, a unit-cell length p, and a laminate thickness t_{lam}. By changing these parameters various corrugation geometries can be realized. The radius R and the opening angle ψ_0 can be calculated from the amplitude and the unit-cell length as follows [12]:

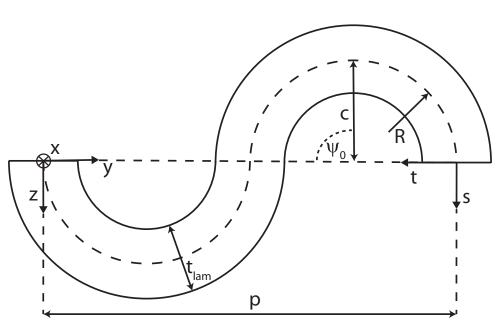

Fig. 1. Definition of geometry of the corrugated structures.

$$R = \frac{16(c/p)^2 + 1}{32c/p}. \tag{1}$$

and

$$\psi_0 = \begin{cases} asin\left(\frac{p}{4R}\right), & c \leqslant \frac{p}{4} \\ acos\left(\frac{p}{4R}\right) + \frac{\pi}{2}, & c > \frac{p}{4}. \end{cases} \tag{2}$$

Fig. 2 shows the geometry of the entire corrugated plate with a size of a times b. All the considered models consist of ten unit-cells or more to ensure that the panel rather than the unit-cell behavior is investigated.

To describe structural stiffness properties of flat laminates we use the following notation of an ABD-matrix as it can also be found in [19]:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon \\ \kappa \end{bmatrix} \tag{3}$$

while for the equivalent stiffness properties of the corrugated laminate we use the superscript $\sim$:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} \widetilde{A} & \widetilde{B} \\ \widetilde{B} & \widetilde{D} \end{bmatrix} \begin{bmatrix} \epsilon \\ \kappa \end{bmatrix}. \tag{4}$$

2.1. Equivalent plate model

Very often corrugated laminates are modeled using homogenized equivalent properties depending on the corrugation geometry and material. These properties can then be used in analytical or numerical models to calculate the structural response of the inner domain solution. In the present study we use the model by Kress and Winkler [12] to calculate the equivalent plate properties of the corrugated structure. The model is valid for thin corrugations consisting of circular sections and cross-ply lay-ups. The equivalent properties are then used in a pre-integrated flat finite element shell model to calculate the vibration behavior of a finite corrugated plate with various boundary conditions. The model is implemented in the commercial FEM code ANSYS using the *SHELL*181 element (a 4 node shell element).

The advantage of this model is its numerical efficiency. In comparison to a detailed model it can only model homogenized structural responses. It is slightly more costly than an analytical model, but it can be used for any kind of boundary conditions and it considers shear stiffnesses $\widetilde{K}_{44}$ and $\widetilde{K}_{55}$ which couple transverse shear line loads with transverse shear strains. The values are set to $\widetilde{K}_{44} = 10^4$ N/mm and $\widetilde{K}_{55} = 1$ N/mm for $c/p > 0.25$ and $\widetilde{K}_{44} = 5000$ N/mm and $\widetilde{K}_{55} = 1000$ N/mm, respectively, for $c/p \leq 0.25$ based on considerations made in [20].

2.2. Finite element shell model

The second model that is used is a finite element shell model where the geometry is modeled in detail. Fig. 3 shows an example of this model with an aspect ratio $a/b = 2$ and an amplitude to per-

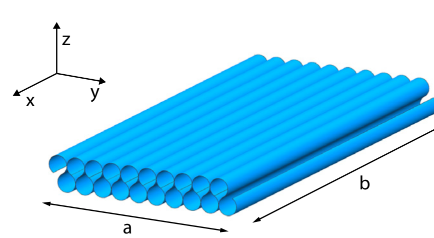

Fig. 2. Definition of the corrugated plate geometry.