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An enhanced reverberation-ray matrix approach for transient response analysis of composite laminated shallow shells with general boundary conditions

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ABSTRACT

In this work, an enhanced reverberation-ray matrix (ERRM) approach is presented to develop an exact and unified solution for the transient response analysis of composite laminated shallow shells with general boundary conditions. The Hamilton's principle and Laplace transforms are employed to deduce the theoretical formulations based on the first-order shear deformation shallow shell theory (FSDSST) and the classical shallow shell theory (CSST). Each of the wave solutions is derived from the exact solutions of governing equations. Under the present framework, the artificial spring boundary technique is introduced to achieve the general boundary conditions. Accordingly, the scattering matrix is modified in unified and compact forms to enable the ERRM approach to deal with all kinds of boundary conditions including the classical cases, elastic restraints and their combinations. Then, the transient responses can be readily calculated by the Neumann series expansion and Fast-Fourier transform (FFT) algorithm. The excellent accuracy, reliability and efficiency of the current approach are validated by several numerical examples. Simultaneously, a comprehensive parametric investigation concerning the effects of elastic restraint parameters, shear deformation and rotary inertia, shallowness, material properties and lamination schemes is performed. Furthermore, the sensitivity of composite laminated shallow shells under different impact loads is also analyzed.

1. Introduction

Composite laminated shallow shells, having small ratio between maximum span and minimum radius of curvature, are extensively used as important structural components in many modern engineering practices requiring high strength-weight ratios and compression-resistance capacity such as naval vessels, transportation, aircraft and civilian industries. It has been shown that such shell structures in these applications are commonly served in complex environment conditions and subjected to many complex loads which may be the arch criminal of the over-vibration and fatigue damage. Therefore, a complete understanding of the transient responses of composite laminated shallow shells under variety of loading conditions is of particular importance to enable safe and economical designs.

Before we handle with the transient response of the composite laminated shallow shell, a brief review related to shallow shell theories should be given first. In general, the three-dimensional (3D) elasticity model with not any assumptions may yield the most accurate response for composite shallow shells [1,2]. However, it is well known that the solutions of the 3D elasticity equations can be obtained only for specific geometries and boundary conditions and in most cases they are impractical. In recent decades, many relatively accurate models have been proposed to reduce the 3D problems to two-dimensional (2D) representations. Most of them stem from the classical laminate shell theory (CST) [3–5], which is known as the simplest equivalent single-layer (ESL) theory and is applicable to thin shells. For thick shells, the CST underestimates deflection and overestimates natural frequency since it ignores the transverse shear deformation effect. To overcome this problem and incorporate the effect of the transverse shear deformation across all layers, the first-order shell theory (FST) has been developed [6–8]. This theory considers the constant shear strain through the shell thickness and only requires the

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C^0-continuity shape functions of the displacements across inter-element boundaries. In addition to these theories, the third-order (TST) [9] and higher-order shell theories (HSTs) [10,11] have also been developed for composite laminated shallow shells. They can eliminate the dependence of the FST on shear correction factors and yield more accurate inter laminar stress distributions. However, in these theories, the higher-order stress resultants are difficult to interpret physically and the computational effort are considerably very large. Therefore, if not necessary, they wouldn't be considered.

During the years, the dynamic analysis of composite laminated shallow shells has been the subject of significant research interest, and numerous research efforts have been made on the basis of various shallow shell theories. The detailed and systematic summarizations can be found in exhaustive review articles by Qatu [12]. Among these researches, the dynamic analysis of composite laminated shallow shells is mostly confined to free vibrations, and a number of analytical and computational methods have been developed. They include, but are not limited to, the closed-form solution [1,2,9,13,14], the Ritz method [15–20], the Galerkin method [21,22],the finial element method [23,24], the boundary-domain element method [25], the meshfree approach [26], dynamic stiffness method [27]. Compared with the free vibrations, the transient response analysis of composite laminated shallow shells has received less attention. Reddy et al. [28] presented a higher-order shear deformation theory (HSDT) to research the dynamic response of cross-ply laminated shallow shell. They found that the deflections and stresses predicted by the classical shell theory (CST) were significantly different from those of the HSDT. Kistler et al. [29] employed the static response characteristics to understand the low velocity impact response of curved laminated panels subjected to transverse impact loads. The effects of boundary conditions, curvature and the validity of linear and nonlinear plate theory have been investigated in their work. In Ref. [30], an analytical method together with the Fourier series expansions and the Laplace transforms were developed for the free vibration and dynamic response of simply supported functionally graded piezoelectric cylindrical panel impacted by different time-dependent blast pulses. A refined 8-node ANS shell element was proposed by Jung et al. [31] to investigate the transient forced vibration behavior of simply-supported FGM cylindrical shells. Based on the first order shear deformation theory, Arachchige et al. [32] used a springs-masses model to predict the low velocity impact behavior of variable stiffness curved composite plates. The effects of the geometrically nonlinear on the transient response of laminated composite shell structures were investigated in [33–36]. Static and transient analysis of moderately thick laminated cylindrical shell panels with various loadings and boundary conditions was performed by Maleki et al. [37] with the generalized differential quadrature (GDQ) method. From the above survey of the literature, it is evident that most of the existing studies on the transient analysis only consider the classical boundary conditions (i.e., simply-supported, clamped, free and shear-diaphragm boundaries) and their combinations. In practice engineering applications, however, the boundary conditions which laminated shallow shells may encounter cannot always be classical and the elastic boundary restraints are much more adaptable. Moreover, the established methods are developed to solve the vibration problems with specific classical boundary conditions, which, therefore, should bring in the constant modifications to the modal function and characteristic parameters to meet the change of boundary conditions. It should further be noted that many of the established methods will lead to very tedious calculations when we want to predict the transient responses of composite laminated shallow shells under arbitrary complex impact loadings. Therefore, developing an efficient method which has the capacity for universally handling with composite shallow shells under general boundary conditions and arbitrary loading cases is of great significance. So far, to author's best knowledge, no work related to this topic has been published.

Recently, a great dynamic analysis technique named the method of reverberation-ray matrix (MRRM) has been proposed by Pao [38] and Howad [39]. Compared with other approaches, the MRRM has better adaptability for different structures since it can trace the wave propagation clearly. Thus, the MRRM has been successfully and fast extended to analyze the vibration responses of space frames [40,41], infinite layered media [42–46], functionally graded elastic plates [47,48], phononic crystals [49,50], finite isotropic structures [51,52] and laminated composite structures [53–61]. However, it is seen that only very few studies are devoted to the transient analyzes of shell structures and no work among them focus on transient response of composite shallow shells with general boundary conditions. Hence, it is worthy of exploiting more potential of this method.

This paper aims to develop an enhanced reverberation-ray matrix (ERRM) approach to fill the noticeable gap in the area of transient response analysis of composite laminated shallow shells under general boundary conditions. Following the first-order shear deformation shallow shell theory (FSDSST) and classical shallow shell theory (CSST), the governing equations of motion in the frequency-domain are deduced by the Hamilton's principle and the Laplace transforms. By arranging groups of boundary springs along edges and endowing them with the corresponding stiffness values, all the classical boundaries, elastic boundaries and their combinations can be achieved. All the wave solutions of shallow shells are constructed by the exact solutions of the governing equations of motion. To present the proposed approach in a unified form, the scattering matrix is redefined by adopting a spring stiffness matrix. The introducing of the Neumann series expansion is to remove any singular points of the original wave solutions, and then the transient responses in time-domain can be accurately computed by the FFT algorithm. In numeric studies, the accuracy, reliability and efficiency of the ERRM approach are proven by the comparison of the results obtained by the current solutions and FEM. The effects of elastic restraint parameters, shear deformation and rotary inertia, shallowness ratio, material properties and lamination schemes on transient responses are also investigated in detail. Moreover, the sensibility of composite laminated shallow shells subjected to impact loads of arbitrary type is analyzed.

2. Theoretical formulations

2.1. The model description

As illustrated in Fig. 1, a composite laminated shallow shell, made of isochoric cross-ply layers, on rectangular platform is considered as the analysis modal. The length, width and thickness of the shell are indicated by L_x, L_y and h, respectively. In the middle surface of the shell, the principal radii of curvature, denoted by R_x and R_y, are assumed as constants. To describe the wave propagation in the shell clearly, we establish a Cartesian coordinate system (o-xyz) with the coordinates x along the length direction, y along the width direction and z along the thickness direction having origin at the middle surface. In addition, from the transverse section of the shallow shell, the kth layer is located between the surfaces $z = z_k$ and $z = z_{k+1}$. With changing the two radii of curvature, shallow shells can have various types of shape. Four typical shallow shells are shown in Fig. 2, these are, plate ($R_x = R_y = \infty$), spherical shell ($R_x = R_y = R$), circular cylindrical shell ($R_x = \infty$, $R_y = R$) and hyperbolic paraboloidal shell ($R_x = -R_y = R$).

As mentioned earlier, the main focus of this paper is to present an exact and unified solution for the transient response analysis of composite laminated shallow shells with general boundary conditions. Thus, before proceeding, we need to clarify the term "general